Promoting an early approach to the algebraic thought in primary and middle school

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Introduction

The international literature in the algebraic learning field underlines a growing crisis in traditional teaching methods regarding this discipline. Various reasons can be found such as: *cognitive (* the jump "in itself" towards generalisation and symbolic thinking, is thought to be difficult) *psychological* (Algebra frightens students who are already frustrated by a difficult relationship with mathematics) *social (*families, and in a more general sense the environment transmit both consciously and unconsciously to children a behaviour that can be defined as mathematics phobia), *pedagogical (*students seem to be less motivated towards studying and thus are more complex to educate) *didactic (*teachers propose outdated and antiquate teaching approaches to algebra). Certainly, each of these reasons interferes negatively with studying algebra, however we feel that the principal cognitive obstacles are situated in the pre-algebraic area (in the sense of Linchevski 1995), and that many *of them arise furtively in an arithmetic context* and therefore pose future conceptual obstacles that are often impossible to overcome during the process of algebraic thinking. Classical studies (e.g. Kieran 1989) underline how students lack appropriate arithmetic structures from which to generalise and how, without the knowledge of arithmetic procedures and their fundaments, students do not possess a conceptual base from which they may thus construct their algebraic knowledge. Moreover, algebra is not usually built through a slow progression like an instrument or object of thought, instead its manipulative mechanisms and computation aspects are emphasised. Consequently, algebra loses some of its essential characteristics: one being the use of an accurate language to describe reality, the other being a potent instrument of reasoning and forecasting, by means of the formulation of knowledge (or hypothesis) of phenomenon (in our case elementary) and the derivation of new knowledge of the said phenomena through transformations allowed by the algebraic formalism.
It is deemed necessary, not only to realize significant changes in teaching algebra to secondary school students but it would also seem necessary to anticipate within primary schools an approach to these changes; beginning with the individualisation of the more productive didactic paths which favour an early approach to algebraic thinking, and simultaneously an intervention on the teachers' conceptions.

Our Hypothesis

As reported in Malara & Navarra (2001), we believe that there exists a strong analogy between the methods of learning spoken language and that of the algebraic language. In order to explain this point of view, we shall employ the metaphor of the *stutter*. When a child learns a language, he or she masters the meanings of words and their supporting rules little by little, developing gradually by imitation and selfcorrection right up to the study of the language at school age at which the child starts to learn to read and reflect on the grammatical and syntactic aspects of the language. Traditionally, in the teaching of the algebraic language, one starts with the study of the rules, as if the formal manipulation should have precedence over the understanding of meanings. The syntax of algebra therefore tends to be taught while overlooking its semantics. The mental models of algebraic thought should rather more be built around that which we might call the initial forms of the *algebraic stutter*. Our own specific hypothesis is that the mental frameworks of algebraic thought should be built right from the earliest years of primary school when the child starts to approach arithmetic thought by teaching him or *her to think of arithmetic in algebraic terms*. In other words, this means constructing algebraic thought in the pupil progressively as a tool and object of thought working in parallel with arithmetic. It means starting with its *meanings,* through the construction of an environment which might informally stimulate the autonomous elaboration of that above mentioned *algebraic stutter*, and then the experimental and continuously redefined mastering of a new language in which the *rules* may find their place just as gradually within a teaching situation tolerant towards the initial, syntactically "shaky" moments which stimulates a sensitive awareness towards these aspects of the mathematical language.

The ArAl Project

The work presented here is part of the *ArAl project – Arithmetic processes to favour the pre-algebraic thought* which our research group set up in 1997 in primary schools[1]. It is aimed at teaching arithmetic with a pre-algebraic key starting from primary schools in such a way as to favour an approach to algebra as a language for the modelling and resolution of problems, even dimostrative ones (Malara 1996, 1999a, 1999b, Malara & Navarra 2000). Principal elements of the ArAl project are: 1) the involvement of teachers both on a professional formation front (discipline and didactic methodology) and in an innovative experimentation front. 2) The planning of the experimental activities within the course of the usual curricular activities. An aspect of great importance is represented in the search for a link between the activities presented by the project and the mathematical curricula for junior schools, with the objective of individualising the ways of obtaining a progressive integration between the two. This aspect is heart-felt by the teachers because, if on one hand it reflects the widespread fear of having to "*make space for algebra"* within a program which is considered ample, on the other hand it also represents the opportunity *for a reflection of ones own knowledge and convictions in the mathematics field so as to arrive at an objective critic of the readings, contents, methods and strategies.*

[1] ArAl is the continuation of a previous project set up in 1993 on these matters but aimed at middle schools (Malara 1995; Malara & Iderosa 1999a). In it are involved nine schools, with 66 teachers and almost 1,500 pupils (1,200 in primary schools and 300 in middle schools). This year it has gained the first position, among more than 600 proposals, in the national S&T competition promoted by the ministery of Education for improving the teaching and learning of Science and Technology in school. It also constitutes one of the Italian contributions to the European ELTMAPS project run by L. Rogers in which are also involved Great Britain, Cyprus and the Czech Republic.

Brioshi and other instruments of meditation within the ArAl project.
A key element in the development of the project is the use of a hypothetical Japanese boy Brioshi: Brioshi is a "virtual" student, of variable age, according to the age of his interlocutor. He doesn't speak the Italian language but he knows how to express himself using a correct mathematical language. Brioshi's Japanese class loves meeting other classes of the same age who are not Japanese, so as to exchange mathematics problems via e- mail. Of course the messages, especially with younger students, can also contain sentences written in Italian or in Japanese (" Dear Brioshi, I want to know if you are capable of…..") which accomplishes – due to the total lack of understanding for the receiver – a marginal function from the ordinary language point of view but which make it powerful and significant in its universality, the use of the arithmetic algebraic code, the heart of the message (represented by the dots) is the mathematical nucleus of the speech. Brioshi has been introduced to all of the classes involved in the project, and he provides a powerful support which often helps to transmit otherwise difficult concepts of understanding for students between the ages of 8 and 14: the necessity to respect the language rules, <u>becomes even more necessary when it is formalised for the strong synthesis of the symbols used</u>
Other important elements for meditation are:

- The analogy with typical situations of natural language (e.g. the multiple representations with which a number may be denoted other than its canonical one, just as each individual may be called with their own name and a myriad of expressions which take into account the person's parents, relatives, friendships, work place etc.,)[2]
- The resource of little "half masks" for an activity regarding the relational aspects of a number.
- The use of a mark or a cloud as a strategy to forecast what is hidden behind a short mathematical text or, to introduce the step towards the use of letters.
- The use of islands, archipelagos and travels as elements of exploration within the number grids which are structured so as favour the synthetic representation of a chain of adding operations, or so as to introduce in an ingenious way the concept of undetermined.
- The use of the scale as an instrument to aid the approach to equations

Some "class scenes"
During the presentation and depending on the time available, significant excerpt of classroom discussion or protocols can be presented to the various students, concerning: 1) Relational aspects of a number and the multitude of its representations (second and third grades); 2) The study of number regularity or patterns (fourth and fifth grades); 3) Problems regarding the translation from the natural language to the

[2] For example, as to the number 12 the writing '3×4' evidenciates that it is a multiple either of 3 or of 4; '$2^2 \times 3$' that it is also a mutiple of 2; '$2 \times 2 \times 3$' brings to '2×6' expressing that it is also multiple of 6 and so on. Analogously in the ordinary language Giancarlo Navarra can be also indicated with the expressions "the father of Alice, "the teacher of mathematics", "the presenter of the report" and so on. In both cases each expression evidenciates a particular property of the subject.

arithmetic-algebraic language and visa versa of interpretation (fourth to eighth grades). We wish also report about the teachers (concepts, behaviour, difficulties, awareness, changes).

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(for a wide reference of the international literature see Malara 1999a)

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