ANALYSIS OF THE TEACHER'S ROLE IN AN APPROACH TO ALGEBRA AS A TOOL FOR THINKING: PROBLEMS POINTED OUT DURING LABORATORIAL ACTIVITIES WITH PERSPECTIVE TEACHERS

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Abstract

In this work we present an activity we carried out with perspective teachers (PTs) during a brief training course aimed at providing them with theoretical and methodological tools useful for the analysis of class processes concerning the development of reasoning through algebraic language. After an outline of the theoretical framework we introduced during the course, we will deal with the problem of the use of theory in the analysis of class processes, highlighting the difficulties faced by PTs.

1. TEACHERS' PROFESSIONAL DEVELOPMENT: THE SIDE OF THE ACTION IN THE CLASS

For reshaping teachers professionalism several scholars stress the importance of a *critical reflection* by teachers on their own activity in the classroom (Mason 2002; Jaworski 2003). Mason, in particular, claims that the skill of consciously grasping things comes from constant practice, going beyond what happens in the classroom, and recommends the creation of suitable social practices in which teachers might talk-about and share their experience. Also Jaworski (2004) stresses the effectiveness of *communities of inquiry*, constituted by teachers and researchers, emphasizing how teachers' participation in these groups helps them develop their individual identity through reflective inquiry.

Our research model is framed in these conceptions, but it also stems from the Italian model of *research for innovation*, which units both an innovation in teaching and a promotion of teachers' professional development. According to this model the interaction between researchers and teachers plays an important role in the training processes in which teachers are involved before and during the experimentation of innovative didactical paths. The key-idea is that research and practice develop in a dialectical process: theoretical results produced by researchers are supported by teachers' practice and evolve through it (Malara&Zan 2002).

In our work, on the side of teachers' professional development, we study difficulties and effects of practices involving collective reflection, identifying categories of behaviour that may be productive for students' conscious learning (Cusi&Malara 2009). Our research experience with teachers made us aware of the difficulties they meet in both designing and implementing socio-constructive teaching. Therefore, we set up and experimented instruments and methods to empower their way of managing whole-class discussions (Malara 2008). Our report takes place in this frame and, precisely, it concerns with the analysis of the role played by the teacher during activities aimed at a renewal in the teaching of algebra, in a perspective that will be outlined in the following paragraph.

2. THE DIDACTIC OF ALGEBRA: OUR MODEL AND THE ROLE OF THE TEACHER

Our vision of the didactic of algebra has developed in a framework in which algebraic language, conceived as a fundamental tool in modelling and in the development of reasoning, is the key-element. Many research studies support an approach to the teaching of algebra aimed at helping students develop an awareness about the role played by algebraic language and the importance of studying it (see for instance Arcavi 1994, Arzarello et Al. 2001, Kieran 2004). Many of them stress the need of devoting more time to activities for which algebra is used as an effective tool but which are not exclusive to algebra (global/meta-level activities according to Kieran's distinction). Referring to the problems related to this particular approach to algebra, Arzarello *et Al.* (2001) stressed that an awareness of the power of algebraic language can be developed only once the student has mastered the handling of some key-aspects that arise in the development of algebraic reasoning. In particular, the authors highlight the use of *conceptual frames* [1] and *changes from a frame to another* and from a knowledge domain to another as fundamental steps in the activation of interpretative processes. Moreover, Boero (2001) argues that *anticipation* [2] is a key-element in producing thought through processes of transformation.

Since we agree with Wheeler's idea (1996) that activities of proof construction through algebraic language could constitute "a counterbalance to all the automating and routinizing that tends to dominate the scene", these kind of activities play a central role in our approach to the teaching of algebra. Therefore we planned and implemented an introductory path to proof in elementary number theory, to be inserted, in coordination with syntactical activities, in the math curriculum of classes of the first biennium of secondary school (grades 9-10). In our experimentations we were able to highlight the difficulties faced by teachers in making their students develop both the fundamental competences for the constructions of proofs through algebraic language and an awareness of the role played by algebraic language during these kind of activities. Therefore we decided to focus on the crucial role played by the teacher during the educational process. Our hypothesis is that teacher's attitudes and behaviours in the class are decisive in fostering (or inhibiting) students' construction of the competences which are necessary for the development of reasoning through algebraic language.

Our research framework about the teaching and learning processes is based on these three fundamental ideas: (1) thanks to the interaction with adults or with more expert peers, the students can activate internal learning processes which help them achieve a higher level of mental development (Vygotsky 1978); (2) one of the main aims of teaching should be fostering, through activities performed in social contexts, a real awareness of the learning process, focussing on the meaning of the actions which are performed in the class (Leont'ev 1977); (3) in order to foster a meaningful learning it is necessary to give students "the opportunity to observe, engage in, and invent or discover expert strategies in context" (model of the *cognitive apprenticeship*, Collins, Brown e Newman, 1989). Giving this opportunity is possible if the teacher is able to bring cognitive and metacognitive tacit processes into the open, trying to make thinking visible. We were inspired by the idea of a teacher that is able to activate in his/her students behavioural processes which are similar to the ones he/she activates in order to identify effective strategies for problem solving. Therefore we decided to focus on one of the possible roles that a teacher could play in the class: the role of model, which is particularly significant especially in the context of activities of proof construction through algebraic language, central in our project. Our research studies made us develop the idea of defining the theoretical construct of *teacher who poses him/herself as a model of aware and effective attitudes and behaviours for students* (Cusi&Malara 2009). In order to make the features of this construct clearly explicit, we will analyze a class discussion, proposed during a laboratorial activity which will be discussed in this report.

3. THE ANALYSIS OF A CLASS DISCUSSION AND THE CONSTRUCT OF TEACHER AS A MODEL OF AWARE AND EFFECTIVE ATTITUDES AND BEHAVIOURS

The following discussion refers to the second phase of our introductory path to proof through algebraic language. The class (10 grade) has already faced activities of translation from verbal to algebraic language and vice-versa. The problem posed to students is the following: "*how can we justify that, if n is an odd number, n^2 is an odd number too*?". In this particular phase, the teacher aims at making students understand the limits of a verbal justification and at guiding them to a conscious use of algebraic language, showing them how to face these kind of problems. During the initial phase of the discussion, two students propose to formalize the hypothesis of this implication through the equality $n=2x+1$. The following excerpt refers to the course of the activity.

1. T[3]: (*addressing A, one of the two students who propose the formalization* $n=2x+1$) How can we convince ourselves that if $n=2x+1$, then n^2 is odd?
2. A: Because an odd number to the second power gives an odd number.
3. T: How can we see this?
4. A: Because odd times odd is odd!
5. T: So here there is the concept of multiplication. (*Addressing the class*) They say: if I multiply an odd times an odd, where do I find factor 2?
6. B: I don't find it.
7. T: So, it is odd.

8. T: And you, Z, how can you see it?

9. Z: Squaring an even number, you get an even. Adding 1 to an even, you get an odd.

10. T: Hold on. Here I read n^2 . Why are you saying "I add 1"?

11. Z: $2x+1$ squared gives an odd number because: $2x$ squared is $4x^2$, then there is plus 1.

12. T: $(2x)^2$ is $4x^2$.

13. T: You say $(2x)^2=4x^2$. $(2x+1)^2$ is $4x^2+1$?

14. Chorus : No !

15. T: Let's get back to what Z says. I can't say that $(2x+1)^2$ is $4x^2+1$. But if I want to convince you that $(2x+1)^2$ is odd, what can I do?

16. O: Let's solve it! (T *writes* $(2x+1)^2=4x^2+4x+1$)

17. T: Now there is "+1" … This quantity here is the problem (*points to* $4x^2+4x$).

18. P: Let's make the total: we take out $4x$.

19. T: Do we really need to take out $4x$?

20. O: It's enough to take out 2.

21. T: Why 2?

22. O: Because then we can highlight an even number, plus 1. (T *writes* $2(2x^2+2x)+1$)

23. Z: But $4x^2+4x$ is the same as $2(2x^2+2x)$!

24. T: Yes, it's the same thing.

25. Z: Ah, I see why! Because taking out 2 you see you get an even. [4]

Let us analyze this discussion from the point of view of both the different roles played by T and the students-teacher interaction, trying to highlight: (1) weaknesses and strengths of the discussion, with reference to the application of *conceptual frames* and *anticipating thoughts* and the coordination between different frames; (2) the role played by the teacher as a "stimulus" to foster an approach to algebra as a tool for thinking, and as a "model" and "guide" in the construction of reasoning. The excerpt can be broken down in three distinct moments: (1) phase of verbal argumentation (lines 1-7); (2) towards a formalization of the property (lines 8-14); (3) proof of the property and reflection upon the importance of choosing a certain representation (lines 15-25).

In the first phase of the discussion A enacts the frame "factorization of a number" to make explicit to the class the justification at the basis of her answer (line 4). Despite her attempt to formalise the answer, A only proposes a purely verbal argumentation. The teacher immediately sets herself in the same frame as the pupil and repeats the reasoning proposed by A to the rest of the class, pointing out the relationship between the fact that 2 is not in the factorisation of n and the fact that n^2 is odd (lines 5 and 7). Through the metaphorical question "where do I find factor 2?", T reminds that 2 is not a factor in the multiplicative representation of an odd number. Though T seems to pose him/herself only on the operative level, neglecting the metacognitive one (there seems to be a lack of an aloud reflection), this particular arithmetical knowledge was already well-established in the class, therefore it can be an implicit assumption in the development of reasoning. T's third statement (line 7), which reinforces A' assertion, seems to block a discussion about the need of a formal proof of the property. Actually, because of the particular moment in the class activity (recollection of students' different point of views), T refrains from intervening in order to pose him/herself as a listener. This fact becomes clear when T invites an other student (Z) to express her reasoning (line 8).

Z's intervention (line 9) is immediately taken by T as an opportunity to introduce the class to a justification of the property based on algebraic formalization. Z, in fact, refers to the additive representation of odd numbers to justify her answer, trying to co-ordinate the frames "even/odd" and "polynomials" while she is trying to 'mentally' manipulate the expression $(2x+1)^2$. Although Z activates a good anticipating thought (she grasps the idea that the objective is to transform the expression until it gets to the form "an even number plus 1"), she faces some difficulties at the level of syntactical transformations, probably because she tries to proceed only verbally. This is a moment in which T must try to *foster* in students *an harmonic balance between semantic and syntactic aspects*. When Z makes an evident mistake in calculating the square of a binomial, the teacher poses him/herself as a *reflective guide*, echoing the student in the form of a question asked to the whole class (lines 12 and 13). Once he/she has amended Z's mistake, T underlines the objective of the syntactic manipulations carried out (line 15) and asks the class to suggest him/her how to proceed. In this case, T is playing a double role: *investigating subject,* putting to the class the question of researching a path suitable to reach the prearranged objectives, and *activator of anticipating thoughts*, clarifying the aim of the activity in order to foster the activation of the "even-odd" frame and the research of the correct syntactical treatment to be performed.

Following O (line 16), T gets to construct the expression $4x^2+4x+1$. At this point, the teacher decides to guide the activity, playing the role of an *investigating subject.* She actually remarks that "+1", Z had mentioned, is in the determined expression, but she points at the remaining binomial $4x^2+4x$ as a "problem to be solved" (line 17). In this way, she lets the class guide the activity, although she remains the point of reference for the discussion. Through this technique, the teacher again acts as an *activator of anticipating thoughts*. After P shows he has not enacted a correct anticipating thought (line 18), T echoes P's proposal, sending it back to the class as a question (line 19). At this point, O enacts the correct anticipating thought, suggesting that 2 might be taken out (line 20). T asks her to justify her idea, so that she can make what she has activated explicit to the whole class. The comment by Z (line 23) shows that the pupil has not interpreted the objective of the manipulation within the frame "even-odd": she actually shows she has not understood the sense of taking out a factor 2 from $4x^2$ and $4x$. T decides to echo her (line 24), simply repeating that the pupil's statement

($4x^2+4x$ is the same as $2(2x^2+2x)$) is right. At that moment Z realises that taking out 2 is a way to make explicit the fact that the expression $4x^2+4x$ is even (line 25). It is important to stress that T always tries not to impose the moments devoted to reflection: her way of repeating students' assertion, also if they are erroneous, and of sending back students' question to the whole class is a clear methodology aimed at *stimulating students' development of reflective attitudes and metacognitive acts*.

This analysis can help the reader clarify some of the definitory elements of the construct of *teacher who poses him/herself as a model of aware and effective attitudes and behaviours* (TMAEAB). This kind of teacher must: (a) be able to play the role of an *investigating subject*, stimulating in students an attitude of research on the problem being studied, and acting as an *integral part* of the class in the research work being activated; (b) be able to play the role of a *practical/strategic guide*, *sharing* (rather than transmitting) knowledge with students, and of a *reflective guide in identifying effective practical/strategic models* during class activities; (c) be aware of his/her responsibility in maintaining a *harmonized balance between semantic and syntactic aspects* during the collective construction of thought processes through algebraic language; (d) try to *stimulate and provoke* the enactment of fundamental skills for the development of thought processes through algebraic language, playing the role of both an *activator of interpretative processes* and an *activator of anticipating thoughts*; (e) stimulate and provoke *meta-level attitudes*, acting both as an *activator of reflective attitudes* and as an *activator of meta-cognitive acts*.

4. LABORATORIAL ACTIVITIES WITH PERSPECTIVE TEACHERS

The activities we refer to in this paper involved a group of 10 new graduates particularly motivated, who still have not worked in school and, waiting for new rules about teacher training courses from the Ministry of Education, expressly have asked to the Science Faculty to organize a brief course propaedeutic to teacher training. As we stressed previously, the training paths for teachers that we usually propose are characterized by a constant dialectic between theoretical aspects and didactical implementation. In the moment we had to work with PTs who still have not had the opportunity to enter in the classes, neither as observers, we faced the problem of a lack in this dialectical relationship between theory and practice. Therefore our main aim became that of giving them theoretical and methodological tools to learn how to interpret their future actions in the classes. The methodology we adopted during this course is strictly connected with this particular situation, but it is in tune with the framework we outlined before. In fact, the activities we performed with PTs can be considered preparatory to the critical reflection they will have to do when, as teachers, they will have to analytically examine their actions to improve the effects of their practice. The course (20 hours) was subdivided into 5 sessions. The activities started with a session devoted to the presentation of: (1) our theoretical framework for the didactic of algebra; (2) the different activities about this theme we realized in the classes, highlighting in particular the experimental path for the construction of proofs through algebraic language (carried out with 9-10 grades students). During the following sessions PTs were involved (sometimes individually, sometimes in groups) in activities of reflection about class practices: we proposed them to analyse excerpts of both class and small groups discussions (produced during our experimentations). Every activity of reflection was followed by a collective discussion aimed both at activating a comparison between PTs and at introducing, not in a transmissive way, theoretical issues about methodological aspects of teaching-learning processes, in order to gradually outline our framework. In this paper we focus on a particular activity of reflection, individually faced by PTs, characterized by the analysis of the previous discussion.

5. PROBLEMATICAL ASPECTS HIGHLIGHTED IN PTS' REFLECTIONS

We asked PTs to highlight: (1) weaknesses and strengths of the discussion, with reference to the activation of conceptual frames, coordination between different frames and activation of anticipating thoughts; (2) the moments in which T plays the role of a TMAEAB; (3) the moments in which T's approach differs from the approach which characterizes a TMAEAB; (4) the (positive and/or negative) effects of T's work on students during the discussion. The analysis carried out by PTs are prevalently line-by-line. Only one PT was able to rationalize local observations in an objective and argued frame about T's attitudes and behaviours. In this paragraph we will focus, in particular, on the problematical aspects highlighted by our study of PTs' protocols, referring to three main aspects: (1) appropriation of the theoretical constructs of reference and their use in performing the analysis of the discussion; (2) interpretation of T's actions with reference to the context (didactical project, particular didactical moment); (3) highlighting of the interrelation between T's behaviours and students' behaviours.

In regard to (1), the examined protocols can be subdivided, referring in particular to the TMAEAB construct, into the following four categories: (a) the PT has interiorized the theoretical construct and he/she is able to refer to it in a pertinent way; (b) the PT recognizes, within the class process, typical components of a TMAEAB, but he/she is not able to identify the specific actions which characterize the highlighted components; (c) the PT only partially recognizes the components which characterize a TMAEAB and he/she does not conduct a punctual analysis of the discussion, making sometimes improper references to theoretical constructs; (d) the PT proposes a naïve analysis of the class process, without referring to the theoretical constructs or referring to them in an improper way.

Most of the protocols belong to the categories (b) and (c): few PTs were able to always correctly refer to the theoretical aspects and appropriately use the specific terminology. Because of space limitations, we present here only some reflections belonging to categories (c) and (d) because they better reveal the difficulties met by PTs in interiorizing the theoretical constructs. For example, many PTs did not realize that the rhetorical question proposed by T in line 5 is aimed at making A's reasoning (line 4) explicit in order to stimulate a moment of reflection. R, for example, observes: "*When T asks 'where do I find factor 2?', he/she plays the role of a* *prompter, inhibiting the anticipating thoughts that could have arisen from students' reflections"*. This reflection testifies a widespread approach used by PTs. They, in fact, often do not analyze T's actions in the context, with reference to what the class already knows and to the particular moment in the didactical path. Their inability of contextualizing the discussion makes PTs interpret as a 'didactical mistake' the fact that T considers obvious that 2 is a factor in the multiplicative representation of an even number. Indeed T's attitude is understandable: aiming at focussing students' attention on the way of developing reasoning through algebraic language, he/she prefers not to re-propose, as a problem, syntactic aspects that most of the students already control.

A similar observation can be done referring to T's choice of quickly performing, without involving students, the syntactic transformation in line 16. The inability of contextualizing this action makes, for example, M assert: *"The teacher is the one who correctly writes the equality $(2x^2+1)^2=$ ...; again he/she is does not make his/her students reflect on the meaning of algebraic expressions and on the equivalence between the expressions at the two sides of this equality"*.

As regards to (1), other difficulties are related to a lack in recognizing, referring to the particular context, some typical features of a TMAEAB. Some PTs, for example, consider as negative T's approach in line 15 because they are not able to recognize that he/she is playing the role of investigating subject. R, for example, states: *"T's attitude of 'I want to convince you' clashes against the meaning of the proving activity"*. An other assertion proposed by T that was not correctly interpreted by many PTs was the one in line 17. Instead of recognizing that T is trying to play the double role of investigating subject and activator of anticipating thoughts, some Pts declare that T is posing him/herself as a mere prompter: *"T suggests the frame to refer to and the quantity on which they have to operate. Therefore he/she is not playing the role of an activator of anticipating thoughts"*(S).

Our study has also pointed out that PTs sometimes propose conflicting interpretations of some micro-actions performed by T. Referring to line 19, for example, we highlighted a contraposition between comments that consider T's approach positive, stressing that T aims at activating a moment of reflection on the meaning of the syntactic transformation to be carried out, and comments that look at T's approach as completely negative, since "*T immediately interrupts the student's proposal*" (D). PTs who propose negative comments to line 19 do not understand that students' bewilderment can be justified because this is one of the first proving activities that the class is facing.

Referring finally to (3), we can observe that only few PTs have tried to highlight the effects of T's action on his/her students. In particular, those who tried to highlight an interrelation between T's actions and students' actions only propose very concise global comments.

6. BRIEF FINAL REMARKS

The comments we presented in the previous paragraph highlight the difficulties faced by PTs both in using theoretical tools to analyse class processes and, in particular, in proposing an analysis of T's actions which takes the particular didactical moment that the class is living into account. Many PTs, in fact, showed to be non completely aware that conducting a balanced lesson on this topic means making students autonomously operate, but also guiding them toward a meaningful learning of how to 'reason' through algebraic language. We think that the limitation of the period of work with these PTs can justify their incomplete assimilation of the theoretical constructs they studied. At the same time, the complete lack of teaching experiences in their careers can be considered an important reason of their difficulties in correctly contextualizing T's actions in tune with the theoretical constructs of reference. However we believe that 'clashing' with these kind of problems could represent for PTs the beginning of a process which could lead to a real professional development. This is testified by the fact that, beyond our evaluation of their protocols, during the following discussions with PTs, they turned out to be very interested in these kind of studies and to really need to go on with the analysis of class processes.

NOTES

1. Conceptual frame is defined as an "organized set of notions, which suggests how to reason, manipulate formulas, anticipate results while coping with a problem".

2. Anticipating is defined as "imagining the consequences of some choices operated on algebraic expressions and/or on the variables, and/or through the formalization process".

3. Here, A, B, O, Z, P and G indicate 5 pupils involved in the discussion while T is for the teacher. Chorus means that the sentence was uttered by a group of pupils in the class.

4. The discussion ends with a further moment of reflection upon the importance of choosing a representation.

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