Unit 4

Matematòca & other mathematical games

2^{nd} – 8^{th} grade

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1. The Unit

This Unit has the aim of supplying, through activities connected to games, some materials allowing the pupils to re/visit arithmetic items under a perspective which favours an algebraic vision. At the same time, with the use of suitable didactic **mediators** (**smudges**, clouds, pieces of paper, etc.) children familiarise with **unknown** numbers and with possibilities to **represent** them. As games go on the materials, which are their actual support, are modified and the instructions written in natural language give place to simple **writings** in algebraic **language**, in which the **letter** represents the score of the dice used in the game.

2. Didactic aspects

Two are the activities composing the Unit: the Game of the little Little masks and the Game of Matematòca. The two games are independent from each other, but the first one can be considered preparatory for the second: the Game of the little Little masks is developed around a guiding thread (a party organised by numbers) that can be enriched and personalised by the teacher; Matematòca is a 'table' game and, as such it has very simple rules (explained in the Unit at the beginning of the Second Phase) to be learnt and respected. The order in which activities are presented is the optimal one, but it is up to the teacher to decide the way to apply them. As a general indication, in both cases three types of situation strongly connected can be identified:
(a) preparatory to the game itself,
(b) inside the game,
(c) problems extending the concepts explored during the game.
Considering the age of the pupils for which this Unit has been set up, and the delicacy of the subject, it was considered as necessary to devote a large space to concrete examples. A big number of meaningful fragments of the class **Diaries** has been included (in fact, this Unit is twice as long as the others). This should be a significant and articulated support to teachers in planning their class activities.
However, it would be important that the faced subjects found a wide and continuous diffusion in the normal didactic activities and didn't remain restricted an isolated experience, although very involving.

3. Priority objectives

- To interpret instructions expressed in natural language (see initial part of 'Unit 1: '**Brioshi** and the approach to the algebraic code');
- **to translate** instructions expressed in natural language into mathematical language and vice versa;
- to get accustomed to a correct and aware use of **mathematical language**;
- to introduce the use of parentheses and of expressions;
- to present the equivalence as a **equivalence relation** (this might be effective, both to acquire symmetry and transitivity of the relation, and to favour the overcoming of the stereotype of operations on the left of the **equal sign** and **results** on the right);
- to favour the exercise of mental calculation;
- to introduce the use of a letter in various meanings (constant, variable, unknown).

4. Enclosures

Two groups of cards for the game of Matematòca are enclosed in the Unit. The first group is formed by 36 cards containing some indications expressed in natural language (to be printed on coloured papers). The second is formed by the same number of cards containing the translations of the indications of the ones belonging to the first group (to be printed on white papers) into mathematical language.

5. Terminology and symbols

Phase	Sequence of situations of growing difficulty referred to the same subject.
Situation	Problem around which individual, group or class activities are developed.
Expansion	Hypothesis of work on a possible expansion of the activity towards an algebraic direction. Its realisation depends on the environmental conditions and on the teacher's objectives.
Supplementary activity	Enlargement towards subjects related to those developed in the preceding **Situations**.
Note	Methodological or operational suggestions for the teacher.
	In the square a problematic situation is proposed. The text is purely indicative; it can also be presented as it is, but generally its formulation represents the outcome of a social mediation between the teacher and the class
Representation	An underlined word in boldface type highlights a link to a subject illustrated in the Glossary.

Square containing the outline of a typical **discussion**; the following symbols may appear:

√	Intervention of the teacher
	Intervention of a pupil
	Summary of several interventions
	Summary of a collective discussion (a principle, a rule, a conclusion, an observation, …)

6. Phases, situations and subjects

All the phases are abundantly supported by significant extracts from the Diaries of activities in the classes.

PHASES	SITUATIONS	SUBJECTS
First	**1 - 6**	Game of the Little Little masks.
Second	**7 - 8**	Propedeutic activities to the Game of Matematòca; Rules for the two versions of the game (second and third class, primary school, 7 – 8 year olds); instructions in natural language and arrow representations; first steps in the game itself.
Third	**9**	Preparatory activities to the Game of Matematòca: the cards in natural language; start of the game.
Fourth	**10 - 11**	Preparatory activities to the Game of Matematòca: the cards in algebraic language; the complete Game.
Fifth	**12 - 13**	Problematic situations arising from the game.

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7. Distribution of the situations in relation to the pupils' age

The distribution represents an indicative proposal based on the experience in the classes taking part in the project. Every situation can be restricted or widened according to the methodological choices or to the opportunities of the teacher.

			Phases and Situations												
			I						**II**		**III**	**IV**		**V**	
School	Age	Cl	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Primary	**7**	**2**	6 hours						5 hours		4 h.				
	8	**3**	4 hours						3 hours		3 hours			5 hours	
	9	**4**									2 hours			5 hours	

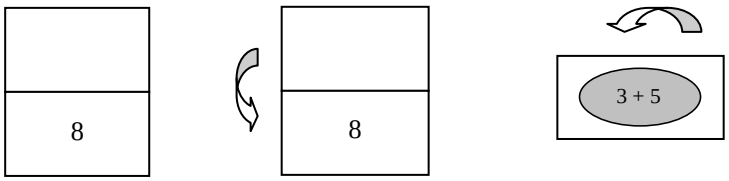
Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

First phase

Game of the Little Little masks

Some pieces of coloured paper are prepared to be fold up in two equal parts.

Fig.1 Fig.2 Fig.3



In the lower part we write a number in **canonical** form (Fig.1) (in the game, this will be the mysterious 'owner' of the little mask). Then we fold up the paper (Fig.2) and we draw a little mask. Inside the little mask, we write a number in a non-canonical form (Fig.3). This second half hides the one containing the number written a canonical form.
The pupils must identify the hidden number; practically, they must recognise the equivalence between different representations of the same number [1].

1. We begin to tell the story.

> The numbers decide to have a little masked party and each of them hides behind a little little mask in order not to be recognised.

We show the little little masks one by one and we ask who are their owners [2].

Diary 1 (second year, primary school, 7 year olds, November)

√ The teacher begins to present some little masks.

3 + 5

«8!»

√ 5 + 8

«11!» «12!» «13!»
√ We invite them to think before answering.
«13!»
√ We ask the pupil to explain how he found number 13.
(rising his fingers) «I counted 6, 7, 8 up to 13»

√ 12 - 7

«I see number 11 underneath» (the child thinks he can make out the number in the hidden part of the little mask, and tries to be clever by guessing).
Number 5 is identified.

[1] *In this phase of start of the **algebraic babbling**, we can also call the canonical form proper **name** of the number, and point out the similarity with the names of people*

[2] *Before starting the activities should take the basic competence of the pupils into consideration, as well as their difficulties, connected to mental calculations most of all, and of the strategies activated to overcome them. Often, younger pupils make calculations helping themselves with their hands, and this slows down the **process**. However, it is advisable to make them **verbalize** the chosen strategy. This accustoms the pupils to give importance to the process besides the **product**.*

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Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

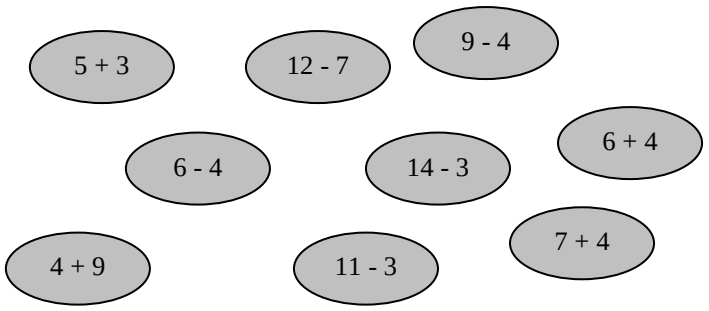
2. The story goes on.

> The party continues and other guests arrive. As it happens for people, some of them have the same name, and can be recognised by their little masks.

The little masks are shown and we ask the pupils to recognise the 'owner' numbers with the same name (different expression of the same number) and to write the identified equivalences on their copybooks. We can agree to call 'brothers' or 'twins' equivalent expressions and their little masks.

Diary 2 (second year, primary school, 7 year olds, November)

On the magnetic blackboard we put these little masks:



√ «Are there any twin little masks?»
💬 «5 + 3 and 11 – 3 hide 8!»
√ «How can we show that these two little masks hide the same number?»
💬 «We do a set to put them together»
💬 «We draw an arrow like this» the pupil comes to the blackboard and writes:

$$5 + 3 \longrightarrow 11 - 3 \longrightarrow 8$$

√ «What do you mean with the arrow?»
💬 «The arrow always means something [3]»
💬 «We put the equal sign!» the pupil comes to the blackboard and writes:
5 + 3 11 – 3 = 8
√ «But how can we make it clear that also 5 + 3 equals 8?»
💬 comes to the blackboard and writes:
5 + 3 + 11 – 3 = 8
√ «But is it true?»
🫘 Pupils verify it and find out that
5 + 3 + 11 – 3 = 16.
√ «And so: how can we make it clear that also 5 + 3 hides 8?»
pupils don't know what to answer. [4]

*[3] This pupil's statement refers to a meaning implicitly **shared** by the class, which remains 'unexpressed'.*

*[4] This diary and many of the following ones clearly highlight the didactic environment in which the algebraic babbling takes place and the 'tolerant' **didactical contract**, which supports it. In this case the concept of equivalence comes through the **metaphor** of 'hiding'. It is developed through some attempts of representation (the reference to sets and, with greater success, to **arrow representations**) to finally grasp the symbol '=', intuition made fruitless immediately after by a misleading observation, which definitely blocks the collective argument led in class with the help of the teacher.*

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

Note 1

The introduction of the concept of 'name of the number' is very important in building the algebraic babbling. In next **Diary 3** we illustrate the ways in which it can be introduced and developed in class.

Diary 3 (second year, primary school, 7 year olds, November)

√ We address a little girl and ask her to tell us her name, the name of some of her family members, her address and so on.
Shortly, these writings appear on the blackboard:

- Viola
- Veronica's sister
- Francesca's daughter
- The girl having 0437 888015 as a phone number
- The only girl in the class 2A, who was born on 24.4.1993
- The girl living in S.Giustina, Villa di Pria number 41.

√ We ask the class to say what all these sentences are. Among other answers, one prevails:
💬 «They are ways of describing Viola»
√ We write number 8 on the blackboard and ask them if they can write it in different ways.
💬 «5 + 3»
💬 «10 - 2»
√ «Could you write number 8 with more than two numbers?» The challenge makes the class enthusiastic.
💬 «1 + 1 + 1 + 1 + 4!»
√ «And could you write number 8 with more than two numbers and with different operations?» The class is more and more involved.
💬 «11 – 1 – 1 – 1!»
An unexpected writing comes out (we are in a second year)
💬 «2 × 4» And, on this track:
💬 «1 × 8»
Children begin to grasp the connection between Viola and number 8.
💬💬 «They are different ways of describing 8»
√ We explain that as 'Viola' is the proper name of their classmate, we can consider '8' as the proper name of this number.
As children have taken interest in the game and they ask to continue it, we propose to write number 9 in a non canonical form using three numbers.
💬 «3 + 3 + 3»
√ «Use different numbers»
💬 «3 + 4 + 2»
√ «Now a more difficult problem: write number 10 using *three* numbers and *three* different operations»
💬 «5 + 5 = 10»
💬💬 The class complains.
💬 «5 + 5 + 0»
√ «Good, but it's always the same operation»
💬💬 Some proposals follow, but all with a single operation, and every time the children are disappointed, because they think they have found the solution. The problem turns out to be complex but challenging; the children don't give up.
💬 «8 + 1 + 1»
💬 «8 + 2 + 0»

(Continues in the following page)

Suitable age related activities	6	**7**	**8**	9	10	11	12	13	**Comments**

→
A little girl's face lightens:«8 + 2 – 0!»
With great satisfaction for everyone, the situation is finally unblocked and proposals become several, although repetitive.
«10 – 1 + 1»
«10 - 2 + 2»
«10 – 3 + 3»
√ «Now try to use four numbers»
«10 – 4 + 1 + 3»
Immediately after, we go back to a more 'normal' (but not at all trivial) proposal.
«10 – 0 + 0»
The meeting ends up with an unexpected 'return to origins":
«8 + 2 = 10» **5**

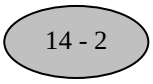
3. The story goes on

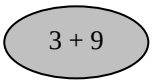
> Since the first party was a success, numbers organise a bigger one, in which they take part, as usual, in a little mask.

If the pupils –as it is likely - have already understood the real aim of the game, we can pass on to an indication expressed in a more precise mathematical way. They have to form groups of 'equivalent' little masks writing the numbers appearing, as they identify them, together with the 'names' of their owners.

Diary 4 (second year, primary school, 7 year olds, November)

At first there are only three little masks:

√ «Do these little masks hide the same number?»
«No!»
Here comes another little mask: 

√ «And now?»
«There are no equivalent little masks»
We add another little mask: 

«Now yes!» The pupil comes to the blackboard and writes:
3 + 9 14 - 2
We continue adding and subtracting little masks.
√ «Who finds little masks hiding the same number, should write them on the copybook»
Some writings:
- 6 – 4 and 10 – 8 hide 2
- 14 – 2 goes with 12
- 15 – 5 and 1 + 9

5 + 7 is 12 **6**

5 *For many pupils (maybe the majority) meeting the operations in the first year, and with their results and their representations determines a strong stereotype, which hinders the correct development of mathematical thought. (i)'3 + 5' o '8 × 2 - 4' are 'operation' and hence (ii) must end with an 'equal sign followed by a number'. Activities such as those in this Unit aim at avoiding the formation of this stereotype blocking, for example, the possibility 'to see' 'a + b' or '2x – y' as numbers*

6 *The equal sign did not appear. We preferred not to force its introduction and wait until the pupils themselves propose it.*

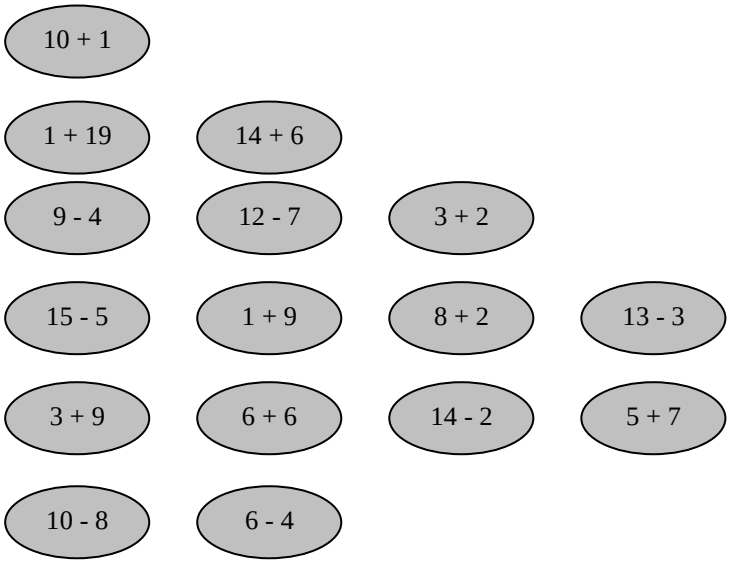
Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

4. The story:

> During the party a game is organised: the twin numbers have to form some rows. Whoever makes mistakes in queuing with the others, being unable to recognise his twins, pays a penalty.

Diary 5 (second year, primary school, 7 year olds, November)

The activity is frantic. In the end, on the magnetic blackboard the little masks are put in these rows:



√ «How can we make it clear that every line hides the same number?»
The children cannot find an answer to this question. Someone proposes to use magnetic counters of the same colour.
√ We remind the activities with Brioshi and then we formulate the problem again. Finally the awaited answer comes.
«We can use the 'equal sign'»
Pupils indicate to put the equal sign where there are *only two* little masks, that is:
1 + 19 = 14 + 6
10 – 8 = 6 - 4
In the other cases, when there are more than two numbers, the class shows resistance to the insertion of more than one '=' and we prefer not to insist [7].

[7] *The two proposed equivalences break the stereotype of the representation of the operation, which sees a single number on the right of the equal sign (see note 5). At this level a sequence of equivalences is still an 'arduous" concept to grasp.*

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

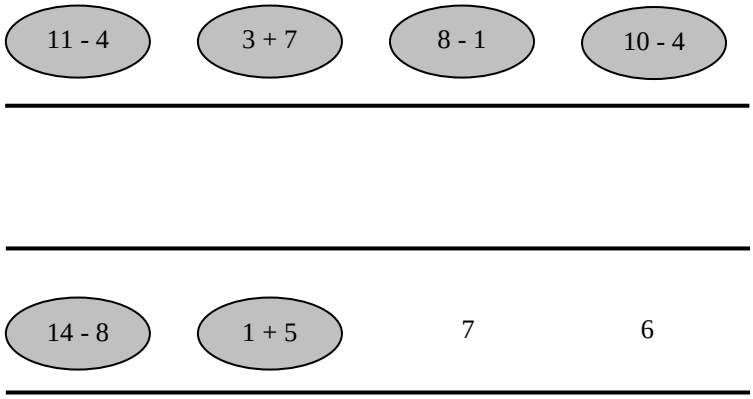
5. The story continues:

> Towards the end of the party, the numbers are tired and some of them already dropped their little masks. In the place where the party was held, there are a few benches. Each bench has been assigned to a group of 'twins'. There are some numbers resting, without controlling if the bench they are sitting on is the right one.

The children must help the careless numbers to find the right place.

Diary 6 (second year, primary school, 7 year olds, November)

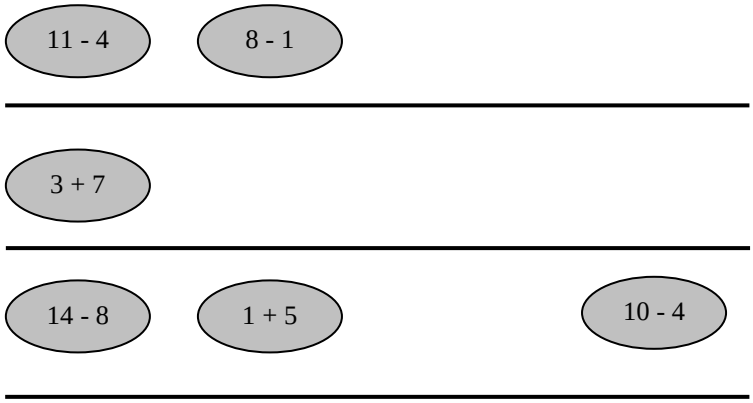
This situation is proposed (the second bench is empty and two numbers are not wearing a little mask):



√ «Is the rule of equal numbers on the same bench respected?»
- «6 and 7 are not the same»
- «10 – 4 goes with 6»

The activity becomes more and more excited. A group of children crowds in front of the blackboard and starts discussing and moving the little masks.
- «1 + 5 goes with 6!»
- «8 - 1 we put it with 7»
- «14 – 8 doesn't make seven!»
- «And so what is there behind 14 – 8?» (he refers to the number 'owner' of the little mask)
- «It is 6 and so we put it on the shelf of 6!»
- «That's no good! There's another little mask to move!»
- «11 – 4 is 7!»

In the end the little masks are placed correctly on the benches:



The activity continues by inviting the pupils to represent the result of their work on their copybooks. Some of the obstacles already met, might reappear. We remind that the didactical contract must be very tolerant. In the following note we linger over this very important aspect.

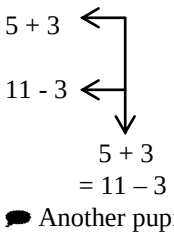
Note 2

Even when the equal sign seems to belong permanently – and soundly - to the patrimony of knowledge shared by the class, new and unexpected didactic sceneries might *obscure* its meaning and consequently put the pupils in the interesting necessity of 're-discovering' it. The following Diary illustrates the way in which second year's pupils build collectively the representation of the equivalence between numbers of the little masks [8].

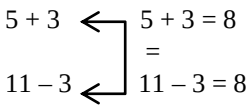
Diary 7 (second year, primary school, 7 year olds, November)

The class is busy with the equivalence between the masks 5 + 3 e 11 – 3.

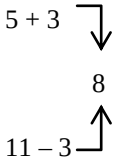
- «They are the same because they have the same result»
- A little girl comes to the blackboard and writes:



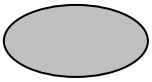
- Another pupil makes some changes:



- A third pupil writes:



- A fourth one draws a little mask and writes beside it:



8 = 8

√ «But isn't there a simpler way to write the equivalence?»

- Unexpectedly a fifth pupil comes and writes:

11 – 3 = 5 + 3

After a little amazement, the class agrees on this writing.

[8] *In several occasions we could verify that a proposal of representation, which is correct as far as mathematical language is concerned, but too precocious, often puzzles the class, not prepared to receive it, and it remains fruitless. If, on the other hand, it is spontaneous and 'attractive', even if the mathematical content is very low, it ends up leading the class towards the* ***collective*** *reaching of stimulating and rich conclusions.*

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Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

6. The game of the Smudge

If the context allows it, we can introduce the approach to the unknown through some mediators, which suggest in a credible and understandable way also for seven-year-old pupils, something that stays 'in the place of' an unknown number. A powerful mediator, whose use will be presented in some Diaries, is the smudge hiding a number.

> At the party somebody found the little mask, which number 13 lost. It is made by the sum of four numbers, but one of them has been covered by a smudge:
> 3 + 5 + + 2

We ask to identify the number under the smudge.

Diary 8 (second year, primary school, 7 year olds, November)

> The pupils formulate some wrong hypotheses.
> A child proposes «10». A rapid test shows that the value isn't right because the sum 2 + 5 + 10 + 2 does not equal 13.
> √ «How can we write that the sum of those two numbers isn't 13?»
> The class cannot answer.
> √ We write the wrong sentence on the blackboard:
> 3 + 5 + 10 + 2 = 13
> then we cancel the equal sign:
> 3 + 5 + 10 + 2 13
> and we ask what symbol we might use.
> The class cannot answer.
> √ We explain that a mathematician would write like this:
> 3 + 5 + 10 + 2 ≠ 13 **9.**
> The pupils show great interest. We propose some other examples and it is evident that they like to apply this discovery.
> The class is very busy with inequalities; the solution to the initial problem is put off.

As we saw in the preceding diary, the smudge can be introduced in the story of the little masked party. It might also be possible to introduce it in an informal way during a normal activity in class on these subjects. The episodes described in the following diaries illustrate two occasion of this type.

9 *It is typical of the jargon referring to operations to use the verb 'to be': 'how much is 16 divided by 2?'. We remind the* operational *value of the verb, which* obscures *(→* ***opaque****) the concept of equality between two numbers. We think that some situations tend to favour this second point of view:*

- *use numbers written in non-canonical form, for example: 6 – 2 = 12 : 3;*
- *use writings formed by several numbers, for example: 15 – 7 + 2 = 13 + 3 – 6*
- *write the number in canonical form on the left of the equal sign, for example: 9 = 15 – 2 – 4*
- *set up 'chains' of equivalences: 3 + 9 = 6 × 2 = 10 + 5 – 3*
- *use the symbol '≠', for example: 7 + 5 ≠ 3 + 10*

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Diary 9 (second year, primary school, 7 year olds, November)

The class is working on an equality of numbers expressed through different representations. On the blackboard this sequence was built:
6 – 2 = 4 = 1 + 3 = 10 – 6
At this stage the teacher proposes a challenge that other classes liked a lot: write numbers equivalent to 4 using more than two numbers.
√ «Now look for a number which begins with 11»
💬 «11 + 5 - 6»
√ «Control the result»
Everybody is absorbed by calculations and find out it doesn't work.
√ «Let's try this: 11 + 5. And then?»
💬 «11 + 5 is 16. If I subtract 6 it's 10 and so …»
√ «So, how much do I have to subtract to 11 + 5 to obtain 4?»
Children find it hard to answer. We decide to recur to a mediator, which had been successful in other situations.
√ «Imagine that someone wrote these numbers on a piece of paper and that an ice-cream smudge fell on the last one. What is the hidden number of the smudge if the whole number is worth 4?» The last number is covered with a cardboard 'smudge':

11 + 5 -

💬 «2!»
√ «11 + 5 – 2. Is it OK?»
💬 «No… it's 14»
√ «Try to count backwards»
💬 «We must subtract 12!»
√ «11 + 5 – 12. Good!»

Diary 10 (two third year classes united, 8 year olds, November)

We reached the conclusion of this work with little masks. Pupils have acquired a good familiarity with the comparison between different representations of a same number.
We decide to examine the class's attitude, when facing an impossible problem for the first time.
√ We propose this situation: find the number which, added to 4, makes the equivalence between these numbers true:
0 + 1 and 4 +
💬 «There ought to be a minus» (the pupil would like to write 4 – 3)
💬 «We don't put the 'equal sign', we put the 'different'» ('≠' met during a preceding meeting).
💬 «In my opinion he got it wrong!»
√ Everyone agrees on this hypothesis; we ask to consider it carefully.
💬 Another child writes on the blackboard:
0 + 1 ≠ 4 + **10**
The bell rings and the meeting is interrupted in a very delicate and stimulating phase. We agree with the teacher that every child will write his own solution. The indication is the following:

> On the second number, after '4 +', there's a chocolate smudge, which hides the solution; compare it with '0 + 1' and explain your conclusions.

10 *A situation like this might be an interesting moment to introduce the letter instead of the number to find.*

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Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

The pupils' **protocols** have been classified and divided in three groups; we report some examples:

A

The authors of this group remain on a cognitive level and identify strategies which, somehow, bring them to the solution of the problem.

(a) Take away the fat smudge to 4 + and if it is 3 you can put it in the set of 7, and 0 + 1 in the set of 1.

(b) 0 + 1 ≠ 4 + because the smudge is equal to 0.

(c) 0 + 1 = 4 + 2 – 5 = 1 **11**

(d) 0 + 1 ≠ 4 + can't be done because 0 + 1 is 1 and 4 + 1 is 5, we can do it only with a minus.

B

The authors analyse the situation from a meta-cognitive point of view; they realise that the problem cannot be solved with an equivalence and they argue their conclusion:

(e) 0 + 1 < 4 + because 0 + 1 is 1and 4 + is bigger than 1.

(f) Because 4 is bigger than 1 and even adding the smudge, the result will always be different from 1 + 0.

(g) 0 + 1 ≠ 4 + because 4 is bigger than 0 + 1 and so it cannot give the same result.

(h) 0 + 1 < 4 + I put the symbol < because whatever number I put after 4 + it will never have the same result as 0 + 1.

C

The authors guess the solution but they don't argue it, or they argue it, but not clearly.

(i) 0 + 1 ≠ 4 + because the numbers are not the same.

(j) 0 + 1 ≠ 4 + because 0 + 1 is different from 4 + .

(k) 0 + 1 ≠ 4 + I wrote different because anything will be different from 0 + 1.

11 *The author of this protocol realises it is impossible to solve the problem as it is proposed and ignores the implicit indication to express 1 through a single addictive couple, thus solving the problem with two operations. This solution can become the object of reflection with the class, showing that there are problems which are unsolvable under certain conditions, but solvable under other ones, or with a reorganisation of the field, as the Gestalt psychology would define it*

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

The Game of Matematòca

In the following pages its Rules are presented in two versions, respectively for second and for third years (primary school). The difference between the two versions depends on the cards used in the game (enclosed in the Unit). The first group of cards contains instructions in natural language and it can be used both in a second and in a third year; the second group contains instructions expressed in algebraic language and it can be used only in a third year. Teachers can personalise the contents of both groups of cards according to their necessities.

RULES OF THE GAME OF MATEMATÓCA - SECOND YEAR'S VERSION (7 year olds)

Materials and rules can be adapted to the class where they are employed.

MATERIALS

- 34 coloured cards with indications written in natural language (enclosed to the Unit)
- a dice
- counters (one for each player or group of players)
- a piece of paper for each player

RULES OF THE GAME

1. On the table we place the coloured cards uncovered, so that to form the frame of a rectangle;
2. we choose one card in a corner as the starting square.
   Every player (or team) in turn:
3. places his counter (from now on referred to as counter) on the starting square;
4. throws the dice once;
5. moves his counter following the indication written on the card up to an arrival square, describing the operation aloud;
6. translates the process (without writing the result) into mathematical language on his piece of paper;
7. waits until his turn.
8. If the result is 0 the player remains on the same square until his following turn.
9. The game ends when a player (or a team) has covered the whole round a fixed number of times.
10. Every player sums up the throws he noted down on his paper during the game.
11. A list is made out and -possibly- awards are granted.

GENERAL RULES

- When it is a player's turn all the others – even his team mates – must not help him.
- If a player interprets his card in a wrong way his mates can help him; if they cannot, the game passes on to the other pupils of the team.

AN EXAMPLE

Throw	Instruction of the card	Score of the dice	Representation on paper
1	*Add 3 to the score of the dice*	*5*	*5 + 3*
2	*Subtract 1 from the score of the dice*	*1*	*1 - 1*
3	*Increase of 2 the score of the dice*	*6*	*6 + 2*
4	*Multiply by 3 the score of the dice*	*2*	*2 × 3*
5	*Sum up the score of the dice to 4*	*2*	*2 + 4*

When he/she arrives to point 9 the player writes (parentheses help in recognising the various scores):

$$(5 + 3) + (1 - 1) + (6 + 2) + (2 \times 3) + (2 + 4) = 8 + 0 + 8 + 6 + 6 = 28$$

We can highlight the identical throws:

$$(5 + 3) + (1 - 1) + (6 + 2) + (2 \times 3) + (2 + 4) = 8 + 0 + 8 + 6 + 6 = 28$$

and write them down as equivalences:

$$2 \times 3 = 2 + 4$$

RULES OF THE GAME OF MATEMATÓCA - THIRD YEAR'S VERSION (8 year olds)

Materials and rules can be adapted to the class where they are employed.

MATERIALS

- 34 coloured cards with indications written in natural language (they only contain additions, subtractions and multiplications by 2)
- 34 white cards with the indications of the coloured cards translated into algebraic language
- a dice
- counters (one for each player or group of players)
- a piece of paper for each player

SUGGESTION

Before reading the rules, it is better to print the cards written in natural language on coloured cards and those written in algebraic language on white cards and use them as a reference while reading.

RULES OF THE GAME

1. Uncovered coloured cards are placed on the table *(we suggest to start with 16 cards)* in order to form the frame of a rectangle;
2. inside the rectangle we put the pack of white covered cards;
3. we choose a card on one corner of the rectangle as the starting square.

Every player (or team) in turn:

4. places his counter on the starting square *(For example: Subtract the score of the dice from the double of 3);*
5. throws the dice once;
6. moves his counter following the indication written in the card, on which he is, up to an arrival square, describing the operation aloud;
7. translates the calculation without writing the result into mathematical language on his piece of paper (*same example: 3 × 2 – d);*
8. picks a white card (for example: (d – 0) × 4);
9. while the other players play in their turn, he tries to identify among the coloured cards in the rectangle the one corresponding to his white one*;*
10. when it is his turn again, if he managed to identify the coloured card, he describes it aloud *(same example: 'Subtract 0 to the score of the dice and multiply by 4')*, he replaces it with the white one and puts the coloured one in front of himself.
11. He continues the game throwing the dice and so on (he does it even if he couldn't identify the coloured card).
12. If the result is 0 the player remains on the same box until his following turn.

As the game goes on the coloured cards decrease, being substituted by the white ones; consequently the difficulties increase, because the white cards are written in algebraic language.

When one of the players arrives on a white card with his counter *(for example: d + 7 – 7*), he:

13. translates aloud its content into natural language *(for example: 'Add 7 to the score of the dice and subtract 7').*
14. If the result is 0 the player remains on the same box until his following turn;
15. throws the dice (for example: 4);
16. performs aloud substitution and calculation and writes it down on his paper (same example: 4 + 7 – 7);
17. moves the indicated number of squares. If he gets the translation wrong or if he cannot do it, he stops for one turn and the rival who can do it gets his score and moves his counter of the corresponding squares.
18. The game ends when one player has covered the whole round a fixed number of times.

CALCULATION OF SCORES

At the end of the game every player:

- translates into algebraic language the sum of the contents of his coloured cards on his piece of paper;
- throws the dice once;
- assigns the value of the dice to the letter and rewrites a new sum;
- finds the result (the higher scores can be awarded).

AN EXAMPLE

The player translates into algebraic language his cards (the parentheses help in recognising the scores):

$$(d + 3) + (d \times 2 – 2) + (8 – d)$$

He throws the dice and number 5 appears; he substitutes this value to the letter d and finds his score:

$$(5 + 3) + (5 \times 2 – 2) + (8 – 5) = 8 + 8 + 3 = 19$$

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

Second phase

This phase presents some activities, which are preparatory for the game itself, but potentially usable independently from Matematòca. The teacher will evaluate the opportunity of doing them all or nor, and when to do them. The spirit of these situations, half way between game, exploration, discovery, collective comparison, discussion, attracts the pupils and makes them willing to share a very active collaboration.
The most significant didactic objectives in this phase are:
a) from the linguistic point of view:
acquire competence in the use of specific terminology of mathematical language;
translate from one language to the other; handle canonical representations (and non canonical ones);
identify equivalences between **paraphrases** (in both languages).
b) from the linguistic point of view:

- favour the comprehension/investigation of the concept of direct/inverse operation with the use of graphs and a metaphorical game, the Game of the Journey;
- favour mental calculation, inducing a reflection on calculating strategies and on the properties of operations and make pupils represent the applied strategy.

The activities are presented in the next three situations:
7. Translations of instructions from natural to arithmetic language and vice versa (activity with coloured cards)
8. Arrow representation of operations (The Game of the Journey)
9. Strategies supporting mental calculation.
7. Some coloured cards are presented to pupils - similar to those to be used in the real game. They have instructions in natural language, in which the unknown does not appear yet (the score of the dice). The activity can be referred to Unit 1: 'Brioshi and the approach to algebraic code'.

Diary 11 (second year, primary school, 7 year olds, February)
(Subject: meaning of 'Add … to …')

√ We ask to translate the card for Brioshi:

Add 1 to 6

«6 + 1!»
«1 + 6!»
√ We discuss on an example: on the wall we see a line of 6 snails, and then another one arrives. How can you represent this situation?
The answer is immediate: «6 + 1».
√ Then we ask them to change the problem of snails so as to adapt it to the writing 1 + 6.
√ The class has no difficulty in answering: 'On the wall there is one snail, and then another six come'.

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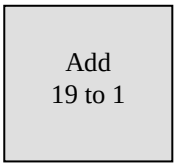
Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

Diary 12 (second year, primary school, 7 year olds, February)

√ We ask to translate the card for Brioshi:



We discuss on the translations of the pupils. The comments are various.
«If you write 19 + 1 and 1 + 19 they are the same thing, only you change place»
√ We highlight that in some lines the equal sign appears, whereas it does not in others.
(speaking of '19 + 1 = ') «It is better to put the equal sign because it is not a real operation [12]»
(speaking of '19 + 1') «If you don't put the equal sign, you don't put the total … seems like the operation is not finished [13]»
«It's like in the game of little masks! Under the little mask a number was hiding!»
√ We congratulate the pupil and we propose to write number 5 in different ways.
These writings are proposed: 4 + 1; 3 + 2; 2 + 3.
√ We want more courage.
The pupils propose: 5 – 0; 11 – 6; 10 – 5; 100 – 95.
√ We propose to put in a multiplication.
«5 × 1».
√ We propose to use two operations.
«3 + 8 + 5».
The classmates point out the mistake.
«2 × 2 + 1».
«3 × 2 – 1».
√ «And in all these cases why did you never write the equal sign?»
«If you put the equal sign you already know what lies beneath!»

12 *The stereotype of the 'real' operation leads back to what was said in note 5*

13 *Some researchers introduced '**lack of closure**' to define this tendency in pupils; the lack of the equal sign grants a sort of incompleteness to writing*

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Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

We can pass on to the opposite activity, proposing some cards written in mathematical language to be translated into natural language, thus anticipating the following cards with instructions written in algebraic language. In next diary we can notice how pupils pass from an initial confusion to the formulation of a correct indication.

Diary 13 (second year, primary school, 7 year olds, January)

We ask how we might translate this card into natural language

6–1+6

In front of some pupils' hesitations, we improvise a game. We invite a girl to go out of the class and we ask her classmates to propose an instruction, which makes the girl do the operation 5 + 3.

One pupil proposes a generic « Write the operation».

As other ideas lack, we decide to write the instruction proposed on a piece of paper and we give it to the girl who obviously does not know what to do. We ask her to go out again and we ask her classmates to elaborate a more precise instruction.

«Write 5 plus 3» The proposal is considered too easy.

«Write the result» the pupils are not really persuaded «It's like 'Write the operation»

«Add 3 to 5».

The proposal is accepted by the class.

The girl is called back in class, she is given the message and she translates it correctly on the blackboard

5 + 3

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

8 The Game of the Journey

The exploration of the concepts emerging from the game of Matematòca requires for the teacher the identification of suitable strategies in order to help the pupils overcome their difficulties. A very effective strategy is that of arrow representation and of the metaphor that we called 'Game of the Journey', which is developed around the dynamic role of arrows to represent operations.
Briefly:
(1) The arrows of the graph represent 'journey out' and 'journey back';
(2) We call 'leaving station' and 'arrival station' the numbers appearing at the ends of arrows;
(3) we can also propose some journeys with 'intermediate stations';
(4) of some 'stations' we know the 'name', of others we must deduce it from the names of the other 'stations';
(5) every 'journey' must be told.
The journeys are various; teachers can decide to introduce them in the game at the right moment, or they can propose them as preparatory activities for the game. Anyway, recurring to arrows represents a very productive support for a *sweet* approach to unknowns and equations. In next Diaries we give some examples of this activity.

Diary 14 (second year, primary school, 7 year olds, January)

√ 'Departure' and 'arrival' are given (the first number is smaller than the second); pupils must find the operators:

7 ⇄ 18

√ «How can we know if in the journey out we add or we subtract?»
💬 «I must look at the numbers: if the first is bigger, I subtract, if it's smaller, I add».

Diary 15 (second year, primary school, 7 year olds, February)

√ Departure and arrival are given (this time the first number is bigger than the second); Pupils must find the operator s_{14}:

8 ⇄ 2

The pupils' proposal:
(a) - 6; (b) + 8; (c) + 10; (d) + 9; (e) + 7.
💬 We comment the proposals and choose the first.
💬 The author of (b) cannot explain «8 doesn't go into 2… »
💬 «I'd have done plus 6»
💬 «No… plus 6 goes below, on the journey back, because from 8 to 2 you subtract, from 2 to 8 you add».

14 *We want to avoid stereotypes like addiction is 'always' the direct operation and subtraction is its opposite.*

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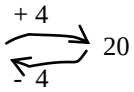
Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

Diary 16 (second year, primary school, 7 year olds, February)

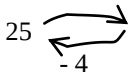
√ 'Arrival' and the operators are given; pupils must find the departure.



🗣 Pupils answer quite easily '16'; we ask how they got this result.
💬 «I put 20 out and I subtracted 4» (he cannot explain why he used the words 'I put out')
√ We ask the authors of this proposal and the other pupils if they can write 'the numbers of the operation' **15.**
They give us the following answers:
💬 «20 – 4 = 16»
💬 «6 + 4 = 10 + a tenth = 20 **16**»
💬 «It's better to do 20 – 4 =»

Diary 17 (second year, primary school, 7 year olds, February)

√ Departure and the operator of the journey back are given, pupils must find the operator of the journey out and the arrival.



√ We ask to explain the graph and to complete it.
💬 «Here it's written '- 4'. We should only write '+ 4' and you get the result»
💬 «If the journey back is '–4' the journey out is '+ 4', so it is 29»
🗣 But some pupils don't accept the solution «It's not 29, it's 21 because 25 minus 4 is 21 and 21 plus 4 is 25!»
🗣 In the following discussion we underline that in the last proposal the direction of the arrow was not taken into consideration.

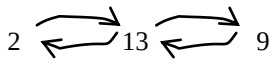
15 *In this and in many other cases, with very young pupils, we consider appropriate – at least in the initial phases - not to insist with instructions like 'represent' or 'write in mathematical language'. It is better to adapt to the 'jargon' of the class ('write the operations', 'write the numbers', 'write the calculations') then passing on to a progressive refining of language*

16 *In this case the pupil uses spontaneously the equal sign in a* ***procedural*** *sense, or better, as an indicator of an intermediate step of the process. The concept of equivalence between numbers totally evaporates*

Suitable age related activities	6	**7**	**8**	9	10	11	12	13	**Comments**

Diary 18 (second year, primary school, 7 year olds, March)

√ We propose a journey longer than the preceding ones: given the departure, intermediate and arrival 'stations', find the operators.
We propose this graph:

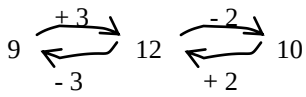


💬 A child completes it correctly and describes his work in a very significant way: «2, to get to 13… plus 11. Instead of 'plus 11' I can say 'I add 11'. Then there is a deceit: 9 is smaller than 13, and so I must subtract. I thought that on the journey out you always added! [17]»

It is better, also with a arrow representation, to elaborate translations in the two directions: from graph to expression and from expression to graph. This activity can open stimulating didactic strategies as for a I approach to unknowns and to the activation of the algebraic babbling.

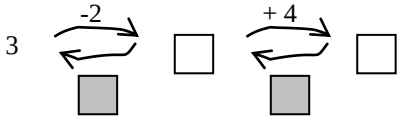
Diary 19 (second year, primary school, 7 year olds, February)

√ We propose to represent this writing with arrows:
9 + 3 – 2 = 10
A pupil comes to the blackboard and with a little help draws the graph (three children agree; the others are confused):

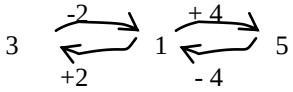


A pupil notices «In this journey I can immediately go from 9 to 10»
With a little help he represents his point of view:

💬 The pupils get to the conclusion that the graph does not express the starting equivalence faithfully.
√ In order to favour the comprehension of the last conclusion, we experiment a new strategy: on the blackboard we stick papers of different colours, substituting numbers or operators and we ask to copy the graph and to complete it writing the numbers under the papers.



Many pupils find the solution on their own.



(Continues in the following page)

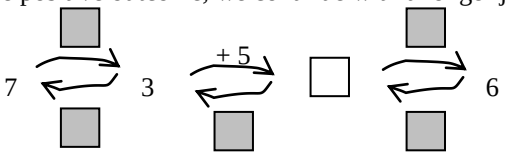
[17] This belief is very common even among teachers, and derives from the mis-concept that subtraction is the opposite of addiction and not also vice versa. If not correctly employed, graphs can favour this stereotype. Therefore, it is necessary to present activities like this, in order to avoid its constitution.

ArAl Project	U4. Matematòca & other mathematical games

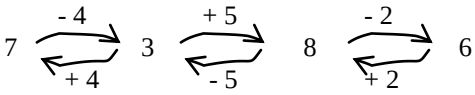
Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

→

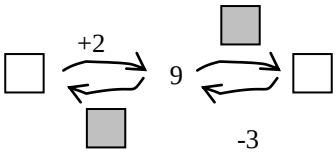
√ Given the positive outcome, we continue with a longer journey:



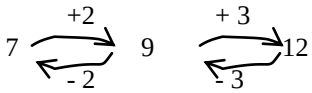
8 pupils out of 12 complete the graph correctly:



√ We propose an even more complex graph, because the unknown is at the start.



Almost everybody complete the graph correctly.



Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

8. Strategies supporting mental calculation

Diary 20 (second year, primary school, 7 year olds, February)

A part of the class has some difficulties in doing mental calculations. We decide to propose some situations that favour this competence.
√ We propose the calculation 34 + 10.
💬 «17» The pupil is invited to explain his calculation. It is clear that he mixed up tenths and units and that in fact he did the addiction '3 + 4 + 10 = 17'.
💬 He explains that he thinks of 30 plus 10 first, and then he adds 4.
√ We try to make pupils represent their classmate's argument. We remind them the work done on canonical and non canonical forms of the numbers and we make them reflect on the fact that their classmate who thought he might add 10 first and then 4, in fact saw 34 not in his canonical form, but in a different form. Together, we get to write on the blackboard:

30 + 4 + 10

💬 The children give the result in a rush: «It's 44!»
√ We try to make them 'slow down' and we make them think over the first calculation their classmate said he wanted to do (30 + 10).
💬 «You exchanged the numbers!»
💬 «You have changed the canonical form!»
√ Together with the class's teacher, we mention the commutative property; some children observe that 'mutation' means 'change'.
All together we reach the following writing:
💬 34 + 10 = 30 + 4 + 10 = 30 + 10 + 4 = 44 **18**
💬 After a little reflection a pupil observes «There are three non canonical forms and a canonical one!»
💬 «We also call them 'little masks'!»
√ We propose a new mental calculation: 25 + 9.
💬 Some pupils organise themselves following a strategy then represented on the blackboard:

25 + 9 = 25 + 5 + 4

√ We invite them to think of another strategy.
💬 Collective activity and resort to fingers slowly lead to these equivalences:

25 + 9 = 25 + 10 – 1 = 35 – 1 = 34

18 *Often teachers want to avoid the chain of equivalences, being afraid that this can favour an incorrect use of the equal sign in the known 'procedural chain', as for example in: 36 – 8 = 28 : 2 = 4 × 5 = 20 (see comment 17). On the contrary, from our point of view it is important to 'bring up a sore point' by stimulating a discussion every time it is necessary. This is the way to build a more stable control on meanings and on the explicit form of the processes. The expansion connected to next diary contains an interesting development of the problems treated in this comment.*

ArAl Project	U4. Matematòca & other mathematical games

Suitable age related activities	*6*	***7***	***8***	*9*	*10*	*11*	*12*	*13*	***Comments***

Diary 21 (second year, primary school, 7 year olds, February)

√ We propose a mental calculation related to a subtraction: 36 – 8.
🫐 Reflection is rich, although confused. Pupils help themselves with their fingers. The majority subtracts six fingers initially and then another two [19]. We get to this writing on the blackboard.

36 – 8 = 36 – 6 – 2 = 30 – 2 = 28[20]

We pass on to individual activities; pupils must try to transcribe the calculations on their copybooks.
√ (a) 9 + 2
🫐 Collective writing on the blackboard, resulting from the comparison among the individual ones:

9 + 1 + 1 = 10 + 1 = 11

√ (b) 7 + 4
🫐 Writing of the pupils: 7 + 3 + 1 = 10 + 1 = 11
√ (c) 22 - 3
🫐 Writing elaborated together with the pupils after a collective discussion and a reflection on calculations with fingers, from the intermediate representation 3 = 2 + 1 and from the final writing (not easy):

22 – 3 = 22 – 2 – 1 = 20 – 1 = 19 [21]

√ (d) 34 – 7
Children are more confident; they prefer to represent the two calculations separately:
(i) 7 = 4 + 3 (ii) 34 – 7 = 34 – 4 – 3 = 27
√ (e) 21 – 9

21 – 1 – 8 = 20 – 8 = 12

Expansion 1

Pupils refer frequently to the activities with the little masks. This suggests an interesting didactic strategy that we might call 'From little mask to parenthesis'.
The initial writing
36 - 8
- being referred to the Game of Little masks- can be represented in this way:

36 - (6 + 2)

Therefore we can propose a different representation (more 'adult') of the little mask, which, in fact leads to a detachment form its concreteness and opens the way to a greater abstractness:

36 – (6 + 2)

The following passage is:

36 – 6 – 2

The situation we propose may not be easy; teachers are invited to experiment it anyway, even in several occasions, but without insisting if the class finds some difficulties in realising the meaning of the passage.

19 *It is necessary to draw the children's attention from the concrete-cognitive level– in which he does calculations by rising one finger and then other eight fingers -to the metacognitive one - in which he 'metabolizes' what he does while he is doing it (he thinks of the signification of the calculations does). This second level is what allows to pass on to a symbolic representation.*

20 *See the* ***Expansion 1***

21 *The most important and delicate aspect is that of maintaining a semantic contact between gestures (rise and bend fingers) and its representation*

Suitable age related activities	6	**7**	**8**	**9**	10	11	12	13	**Comments**

Third phase

We pass to the game of Matematòca itself; we start with coloured cards containing sentences written in natural language.

Note 3

Matematòca is appreciated by the pupils but, like any game, it is necessary to interiorise the Rules and go beyond the initial phase. Some remarkable differences between second and third year pupils emerge from the following points of view:

- psychological (confusion in front of unusual instructions);
- behavioural (intolerance towards slower classmates, impatience, loss of concentration);
- of the working method (scarce autonomy, often chaotic commitment, need for requests and suggestions, difficulty in using the correct answers of the team mates, tendency to give an answer and then be distracted, lack of respect for the rules of the game);
- of reading (often difficult);
- of language control (difficulty in comprehension of the logic mathematical content of instructions, weak knowledge of the terms of mathematical language, little familiarity with **paraphrases** of the operations, stereotyped use of names of the signs to define operations);
- mathematical (difficulty in mental calculations, perplexity in identifying operations, doubts with direct and inverse operations, confusion between addictive and multiplicative concepts, doubts on 'double' and 'triple' [22]).

If, as it often happens, during the game the teacher tends to favour the operative direction (tell the *result* of the calculation after throwing the dice) then the procedural aspect prevails over linguistic reflection and pupils tend to concentrate on the product ignoring the explicitation of the process. The teachers' preoccupation – above all in the first years of primary school – of making the pupils understand algorithms and making them acquire the necessary automatism is understandable, but in many occasions it happens within a too 'closed' didactic contract. As a consequence, the teacher risks dependence on a reassuring *jargon* ('the teacher *knows* what I mean even if I don't say it or say it badly') which ends up being *weak*. For example: two paraphrases of subtraction made explicit with the words 'take away' and with the classical 'minus one', can be accepted, but they must be accompanied by a reflection highlighting the different meaning. The first is the verb indicating the process, the second is the name of the symbol, which represents it.

Some observations on the Diaries presented in the Unit.

The *personality* of a game (and also the Matematòca) shows itself during its development, as it is tightly connected to its *dynamics*. It is difficult to summarise it through the description of situations, as in the diaries, *fixed* in time. The only way to understand the game is, then, *to play*. Obviously, in our case, the personality is mainly expressed through the richness of mathematical hints offered by Matematòca and the activities that they can stimulate. Diaries represent the attempt to illustrate some aspects of this personality through some significant photograms that are, at the same time, *evidence of* episodes happened in real classes during the game and *prototypes* of class dynamics and of the pupils' attitudes. They record activities performed in three second years and in four third years.

[22] *In this case difficulties are probably also due to the overlapping between the addictive (ex: 3 + 3) and multiplicative concept (3 x 2), that create a sort of short circuit in the pupils, above all in the first phases of learning of the multiplication. It is significant that in a second year, when asked «what does 'Double mean?» a pupil answered «The symbol 'plus'»; only after several attempts another pupil explained the term with «put it twice».*

Suitable age related activities	*6*	***7***	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

9. The Game with coloured cards

We start the game only with the coloured cards. As pupils get acquainted with the rules, we gradually introduce (in the third years) some hints for reflection on translation from natural into algebraic language and vice versa, thus preparing the introduction of the white cards. Initially, it is better to favour group activity.

Diary 22 (second year, primary school, 7 year olds, December)

The counter of a child is on the card

Multiply by 4 the score of the dice

The child throws the dice and the result is 5.
√ We ask to translate the content of the card after throwing the dice for Brioshi.
Translations: (a) 20 (b) 4 + 5 (c) 5 × 4.
«20 is an entire number, whereas 4 × 5 are two number and if you put them together, they give 20»
«4 × 5 is the little mask and 20 is the proper name»
This last remark unexpectedly refers back the activities developed some months before with the 'proper name' of a number and the little masks.

There is another counter on this card:

Add 3 to the score of the dice

Throwing the dice the come out number is 4.
√ As usual we ask a translation for Brioshi.
Correct translation proposed by several pupils: 4 + 3 = 7.
The following player takes this card:

Subtract 1 to the score of the dice and add 6

We throw the dice and get 6.
√ We ask the translation.

(d) 6 + 6 ('evaporates' the subtraction)
(e) 6 – 1; ('evaporates' the addition)
(f) 6 – 1 + 6 (only 2 pupils).

Suitable age related activities	*6*	***7***	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 23 (third year, primary school, 8 year olds, January)

We are playing for the first time. The activity is collective and children take part in it with great interest. There are no great difficulties.
One counter is on the card

Subtract the score of the dice from 8

√ «How can we translate this card for Brioshi?
🫘 Translation written on the blackboard after a comparison: d – 8.[23]
√ We propose a paraphrase of the sentence: 'Subtract from 8 the score of the dice'.
🫘 The discussion leads to the correct translation: 8 – d.

Diary 24 (two third year classes united, 8 year olds, February)

A pupils arrives on a very peculiar card:

Multiply by 0 the score of the dice

√ We ask to interpret it and translate it for Brioshi.
🫘 «d × 0»
√ «If d is 1 how far do you move?»
Pupils realise that they would stand still.

23 *Here is a mistake known in the related literature as 'reversal error', owed to the confidence, in translating, in the order in which term appear in the text (in this case, 'the score of the dice' precedes '8').*

ArAl Project	U4. Matematòca & other mathematical games

Suitable age related activities	*6*	***7***	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 25 (two third year classes united, 8 year olds, February)
It might be convenient to favour comprehension using the arrow representation.

One team is on this card:

Add 2 to the score of the dice

√ We make them interpret and translate it.
(without any difficulty many hands are raised) «d + 2»
The following team has its counter on the card:

Add 2 to the score of the dice and then subtract 1

√ We ask to translate it into mathematical language.
(without any difficulty) «d + 2 - 1»
√ We stop the game and propose this problem:

> Imagine you throw the dice without showing the score. Your classmates see you taking 5 steps. How can we represent this situation in mathematical language so that Brioshi can understand what happened? **24**

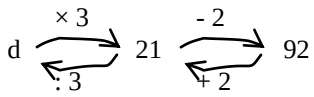
«d + 2 – 1 = 5»
«d is 4 because 2 - 1 is 1, and so 5 – 1 is 4»
√ As we had done in other classes, we ask to tell the story with a graph.
We propose the sentence:

Multiply the score of the dice by 3 and subtract 2.

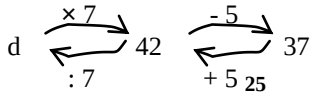
Pupils propose: 'd × 3 – 2'.
√ We propose the game of the child again. this time he takes 10 steps.
Pupils propose:

(d) 4 × 3 – 2 = 10 (e) 3 × 4 – 2 = 10 (f) d × 3 – 2 = 10

The two paths are built:



We find out that d is 4.
√ New game: the player has this card: 'd × 7 – 5 = 37'.
A pupil builds the following graph:



They find out the value of d.
√ We ask to make it explicit, pronouncing it.
«The score of the dice is 6»

24 *In some of the following Situations and in the Expansion of page 35 several problematic hints will be proposed, which might enrich the development of the game*

25 *As was stated in the preceding note, we can introduce division in the departure journey, in order to verify if multiplication is identified easily or not.*

Suitable age related activities	*6*	***7***	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 26 (second year, primary school, 7 year olds, January)
We are inventing new cards for the game
(Subject of the discussion: meaning of 'to double')

√ We propose to invent new cards for the Game of Matematòca.
Proposals are very doubtful.
√ We remind them that every card referred to the number on the dice.
Slowly some proposals begin to appear.
«Double by 1 the number of the dice»
√ We ask the author of this sentence the meaning of the term 'double'.
«I do double the number of the dice»
«No! Double means add 1!»
√ We express amazement «In your opinion 'double' and 'add' are the same thing? If the dice gives 4, what's the meaning of doubling the 4?»
«4 multiplied by 2 plus 1!»
√ «Well, what's the double of 3, then?»
«The double of 3 is 6»
«No! It's 3 by 3!»
«No! It's twice 3»
√ We ask them to explain the meaning of 'double', 'triple', 'quadruple' and we mention 'quintuple', 'sextuple' and so on. Then we go back to the initial sentence 'double the number of the dice' and we ask what it means.
The answer is unanimous 'Multiply by 2 the score of the dice'.
√ We try to verify this «How much shall I move the counter on this card if I throw the dice and the result is 3?»
«3 + 3»
«3 × 2 = 6»

Diary 27 (third year, primary school, 8 year olds, January)

√ We explain that at the end of the game, every player will have to sum up the scores of his cards. We show some and ask them to translate them into mathematical language and to represent their sum.

Multiply by 0 the score of the dice	Subtract the score of the dice from 8	From the score of the dice subtract 1	Double the score of the dice and subtract 1

Pupils propose this writing:

$$d \times 0 + 8 - d + d - 1 + d \times 2 - 1$$

√ We suggest to introduce some parentheses in order to highlight the content of every card.
Pupils change the preceding writing:

$$(d \times 0) + (8 - d) + (d - 1) + (d \times 2 - 1)$$

√ Now we imagine we throw the dice and get 5. We ask them to substitute the value 5 to 'd' and to find the final score:
Pupils dictate:

$$(5 \times 0) + (8 - 5) + (5 - 1) + (5 \times 2 - 1) = 0 + 3 + 4 + 9 = 16$$

Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Fourth phase

Introduction of the white cards (translation from algebraic into natural language)

10. Before beginning the real game, it is better to check the comprehension of letter 'd', by presenting single cards and asking to interpret them.

Diary 28 (third year, primary school, 8 year olds, January)

√ For the first time we present one by one some white cards and we ask the pupils to interpret their meaning.

3 × 2 -d

- «3 by 2 is 6 … minus … 2, maybe»
- «Letter 'd' means 2»
- «No, 'd' means 12 **26**»
- «'d' means 'dice'»
- «The number that resulted from the dice»
- «'d' means you skip one turn»
- «No, 'd' means 12»

√ We propose a new card **27**:

d - 1

- «'d' means you skip one turn **28**»
- «I think I have to go back of one box»

√ We propose a new card:

d + 3

- «We have to add 3 to the number of the dice»
- «From the score you have obtained go on for three boxes»
- «Add 3 to the score of the dice»
- «Add up 3 to the score of the dice»
- «Sum up 3 to the score of the dice»

√ We propose another card:

d –1 + 6

(Continues in the following page)

26 *These are very common interpretations: the letter in the card is still seen as 'a letter of the alphabet' and it is within this context that its possible meaning must be looked for. In this case 'd' is interpreted as the initial letter of the terms 'two' and 'twelve' (both beginning with a 'd' in Italian)*

27 *It is a productive strategy: even if the pupils' interpretations are still far from the correct one, we propose some situations similar to the preceding one; as will be clear also in this case, discussion and reflection allow to 'polish up' the various interpretations until the correct one is reached.*

28 *Probably the pupil imagined that the dice gave a 1, he calculated the result mentally (0) and perceived it as a 'pause'*

Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

→
💬 «Subtract 1 plus 6 from the score of the dice»
💬 «From the score of the dice subtract 1 plus 6»
💬 «From the score of the dice subtract the sum of 1 plus 6»
√ We go back to the use of parentheses. The teacher reminds the pupils that they called them 'little houses' **29**.
💬 The pupils copy on the blackboard the writing that would mirror these definitions: d – (1 + 6). Then they conclude that in the original card «we need to subtract 1 and add 6, in the second one we need to subtract the sum of 1 plus 6»
√ We propose another card:

d – 6

💬 «From the dice take away… subtract 6»
√ Another card:

8 – d

💬 «Subtract 8 from the score of the dice»
√ We ask if the writing '8 – 3' corresponds to: 'Subtract 3 from 8' or 'Subtract 8 from 3'.
💬 «It's true! Subtract the score of the dice from 8»

29 *Once again we experience the opportunities and richness of a metaphorical teaching, powerful in its freshness, above all when metaphors are invented by the pupils or elaborated with their decisive participation (about this, see* ***Expansion 1****).*

Diary 29 (third year, primary school, 8 year olds, January)

√ We propose a white card and ask the meaning of the writing.

d × 2

💬 «That you have to find out the number and then, for example, multiply by 2»
√ «And how do I find it out?»
💬 «You must use the dice»
💬 «You must throw the dice and read the number»
we throw the dice and get 2.
💬 «2 multiplied by 2»
💬 «That we need to add 4!»
💬 «4 multiplied by 2 because d is the fourth letter of the alphabet' **30**»
💬 «It can also be 2000 multiplied by 2»
💬 In the discussion the meaning of the card is **negotiated**.

30 *See Comment 24.*

Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

11. We can test the ability of the pupils in comparing several cards. The comparison allows to explore paraphrases and find equivalences.
It is interesting to compare the next two diaries, in which the same situation is analysed by different classes.

Diary 30 (third year, primary school, 8 year olds, January)

After the pupils have told the games they played since our last meeting, I put five cards on the table:

d + d	d - d	2 × d	0 × d	d × 2

√ «In your opinion are there any cards where you would take the same number of steps?»
🗣 The first, the third and the fifth card are identified.
√ «How can we represent the fact that I take the same number of steps in mathematical language?»
🗣 The first proposals are very naive: represent with 'downwards arrow'. Nonetheless, they get to this writing:
(a) d + d = 2 × d = d × 2
A second equivalence is identified:
(b) d - d = 0 × d
√ When asked to explain this equivalence:
🗣 «Because for example 1 minus 1 is 0, and 0 multiplied by a number gives 0»
A little girl comes to the blackboard and writes - replacing 1 with 'd' in (b):
💬 «If I do: 1 – 1 = 0 × 1 = 0»
Another proposal is wrong as regards the problem, but it raises a very interesting issue about relative numbers:
💬 «You can also do number 4 minus 5 which gives … -1 but then… »
√ We ask if it is possible to obtain the situation '4 –5' from a card with the writing d – d.
💬 «'d – d' means 'from the number of the dice subtract the number of the dice'. Dice minus dice cannot give dice minus 5… they must be identical digits»
🗣 Some children give examples of what they mean by substituting 5 to d in (a). They obtain:
(c) 5 + 5 = 10 (d) 2 × 5 = 10 (e) 5 × 2 = 10

Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 31 (two third year classes united, 8 year olds, January)

√ We put these five cards on the table:

d + d	d - d	2 × d	0 × d	d × 2

√ «The counter reaches these cards. In your opinion are there any cards that say the same thing?»
Several pupils identify d – d and d × 0. A child proposes the writing:
d – d = d × 0
Other pupils identify d + d e d × 2.
√ We ask to represent this situation for Brioshi. A girl proposes:
d + d = d × 2
«It doesn't work, because if we have 6, d plus d is 6 plus 6, and 2 multiplied by d is 2 by 2»
Pupils are doubtful but they don't know how to reply.
√ «Let's imagine that a team has its counter on 'd + d', that they throw the dice and obtain 5. How far do they move?» The answer is immediate:
«10 steps!»
√ «And if the other team has its counter on 'd × 2' and obtains 5. How many steps do they take?» The answer is less immediate:
«10 steps again!»
The classes agree that the two writings are equivalent.
√ «How many steps does the counter take, being placed on this card?»

d+5-5

«We need to know the value of the dice!»
(addressing the child who spoke before) «It can be 1, 2, 3, 4, 5 and 6!»
After a few seconds, a child exclaims
«But it always gives the score of the dice!»
The classes agree on this conclusion.

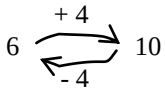
Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Fifth phase

12. As the pupils' competence in translating from natural into mathematical language and vice versa increases, we can try to propose more problematic situations using Brioshi as a mediator. Once again a significant help comes from the resort to arrow representation, which can be a support for an opening towards letters as unknowns and towards equations. This aspect is very delicate, and teachers should deal with it with great attention. The next diaries show how things went in some classes of the project.

Diary 32 (third year, primary school, 8 year olds, January)

√ We start by making the class describe the games they invented after our last meeting, based on the cards of Matematòca.
Then we propose an interpretation/translation activity of sentences written on the cards in mathematical language.
(a) (n × 0) + 1: 'I multiply a number by 0 and then I do the addition'.
(b) (0 × n) + 4: 'I multiply 0 by my number and then do plus 4'.
(c) (n - 0) + 4: 'I subtract 0 from the number and then do plus 4'.
(d) (n + 2) - 1: 'I add n to 2 and then subtract 1'.
💬 We discuss on the correctness on this last translation.
As the competence in interpreting the instructions containing a letter is very high, we propose a more complex writing; we imagine that Brioshi sent it and we must interpret it:
n + 4 = 10
Many pupils are doubtful. In the end, some proposals for interpretation are proposed.
💬 «Brioshi asks what number is under n»
💬 «What number is hidden under it»
It is evident from the childrens' attitude that they understood that 'under n' there is 6.
√ «But what does the sentence n + 4 = 10 mean, in your opinion?»
In front of the class's doubts -although very open to collaboration- we perform a representation of the sentence with a graph, which the pupils recognise even if the teacher declares she didn't spend much time on the subject. The letter n disappeared; the graph is 'arithmetic':



√ We propose this problem:

> On a piece of paper we find the writing: '(n + 2) – 1'. We don't know which number n is, but we have another piece of information: if I do the calculation, I know I will obtain number 20. Can you find number n?

💬 After a few seconds all the children wave their arms. They propose the writing:

$$(n + 2) – 1 = 20$$

They understand that in the place on n there is 19. We write all together:

$$(19 + 2) – 1 = 20.$$

The calculation allows to show that the answer is correct.

(Continues in the following page)

Suitable age related activities	6	7	**8**	**9**	10	11	12	13	**Comments**

→

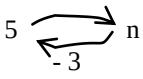
√ We make it clear that what really matters is not finding number 19, but explaining -for example to the children of other classes-the way to find number 19. Can anyone explain his argument? **31**

💬 «I try all the numbers to see what comes out»

💬 «We choose a random number smaller than 20 and do plus 2 and minus 1»

💬 «20 minus 1 gives 19 plus 2 gives… 21 and then minus 2» The classmates observe that the 'minus 2' has nothing to do with it.

√ In order to favour comprehension, we propose a representation of a situation with a graph:

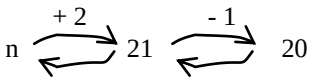


We ask to find the value of n.

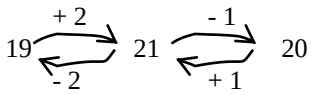
💬 Some pupils propose 8 because 5 + 3 = 8.

💬 «Yes, because we know it's the opposite; usually, if underneath there is '-3', above there is '+3' and so it's 8»

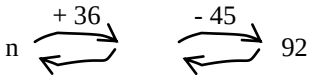
√ We ask to represent the initial sentence (n + 2) – 1 = 20. Slowly, with the help of all the children we get to this writing:



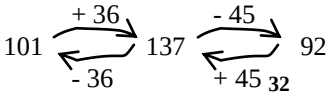
√ We ask how we can complete the graph. Together with the children we get to this writing:



√ We propose another example: (n + 36) – 45 = 92.



💬 The pupils complete the writing starting from 92 and with calculations they find the value of n.



32

31 *Once again we point out the difference between 'product' and 'process'.*

32 *With older pupils we can also propose graphs with a multiplication; with younger pupils we have to be sure, before doing it, that there are no difficulties in identifying the division as opposite to the multiplication. Anyway, the stimulus might also be productive to consolidate this concept.*

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Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 33 (two third year classes together, 8 year olds, February)

√ The class is interpreting some cards written in formalised language. We start with

10 - d

💬 «Subtract 10 from the score of the dice»
💬 «Take away 10 from the score of the dice»
√ «Do you agree?» The pupils add two further sentences:
💬 «From 10 take away the score of the dice»
💬 «Take away the score of the dice from 10»
√ «Are these sentences equivalent? Do they say the same thing?»
🗣 Some pupils agree on the first two ones.
√ We continue with the classes to think of the difference between the two types of writing, because it is evident that, apart from a few of them, the others cannot grasp the difference. Here we find realised the situation connected to the sentence 'take away 10 from the score of the dice' and we ask the pupils to translate it into mathematical language, imagining we throw the dice and obtain 6.
🗣 Some pupils translate '10 – 6'. Many others agree, but immediately after some others realise that the correct translation would be 'd – 10', which is different from the initial writing.
💬 «It isn't right, because if we get 6 we can't do 6 – 10»
√ «And so how do you read the sentence 10 - d?»
🗣 «Subtract d from 10»
💬 «The subtraction doesn't have the commutative property!»
We praise this pupil.
We consider it useful to propose the comparison between two diaries on the same problem. The reactions are very similar.

Diary 34 (two third year classes together, 8 year olds, February)

√ We decide to propose this situation:

> Another class is playing Matematòca. Two teams have their counters on two cards and they send us this message: $d - 1 = d \times 0$. In your opinion, is it possible to understand what the score of the dice was?

After a pause, a little girl says:
💬 «If the dice gives 1 it's possible»
√ We praise the girl and ask how we might write down this conclusion for Brioshi. Obviously, the classes are very doubtful.
🗣 Together, we reach this writing:

if d = 1 then the cards are equivalent

which is then refined by the pupils into:

if d = 1 then $d - 1 = d \times 0$

√ «And if this happens? Mind what I write now: $d \neq 1$»
🗣 Shortly after, the pupils dictate:

if $d \neq 1$ then $d - 1 \neq d \times 0$

Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 35 (third year, primary school, 8 year olds, March)

√ Given the ability of many children to think over writings expressed in symbolic language, we try to detect how far we can 'carry things'.
We propose this problem:

> What number should come out throwing the dice so that the counters that are on these two cards take the same number of steps? And we write on the blackboard:
> $d - 1 = d \times 0$

The class reacts very well.
«We need a 1»
Shortly we get this writing together with the children:

$$\text{if } d = 1 \quad \text{then} \quad d - 1 = d \times 0$$

√ We ask to reflect on what happens if any other number comes out.
The children try some possibilities and find out that the two cards don't give the same results anymore.
«If d is worth from 2 to 6 it is different»
proposes to write $d - d \neq d \times 0$ (the others understand it is wrong).
«If the sign is different, well then it might be 6» and he proposes:

$$6 - 1 \neq 6 \times 0$$

√ We try to draw the pupils' attention on the fact that we can get to a relation between the two quantities, other than a simple inequality. We make to pupils of different heights stand one beside the other, and the class notices easily that 'one is taller than the other'. The same conclusion is more difficult also because, probably, the hypothetical thought comes into play. We ask to examine what happens if the value is 2 and to remember what we said about the two childrens' heights.
Some pupils discover that if d is 2 the first card gives a bigger number of cards than the second one. On the blackboard we write in formalised language:

$$2 - 1 > 2 \times 0$$ **33**

We do the same thing for all the values:

$$3 - 1 > 3 \times 0$$
$$4 - 1 > 4 \times 0$$
$$5 - 1 > 5 \times 0$$
$$6 - 1 > 6 \times 0$$

We help the class in the passage to the general case (the thing happens without any difficulty):

$$d - 1 > d \times 0$$ **34**

33 *It's not difficult to see in this activity a possible expansion on inequalities.*

34 *Obviously in this case 'd' has a very concrete value: the score of the dice and it varies in a very small set.*

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Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

Diary 36 (two third year classes together, 8 year olds, February)

√ We propose this situation:

> A child has his counter on the card 'Add 2 to the score of the dice'; he throws the dice but does not show the score he got. His classmates see that he takes 7 steps. Knowing that the child behaved honestly, can we understand the score of the dice?

(without any difficulty) «It's 5!»
√ We ask how this situation might be described in mathematical language. We receive three proposals:
(a) 5 + 2
(b) d + 2 + 5
(c) d + 2 = 7
Comparison and discussion allow to understand that sentence (c) is the correct one.

Diary 37 (two third year classes together, 8 year olds, April)

Comparison between numbers expressed in algebraic language

√ We challenge the class: «As you are so good, compare these two numbers» I go to the blackboard and write:
d and d + 1
The pupils don't know what to answer.
√ We propose the comparison between 7 and 4.
The answer is immediate:
7 > 4
√ We propose the comparison between 3 and 6 + 2.
Once again the answer is immediate:
6 + 2 > 3
√ We propose 5 and 5 - 1.
This time we get three proposals:
(a) 5 = 5 + 1 (b) 5 > 5 – 1 (c) 5 ≠ 5 - 1
The discussion leads the pupils to realise that (b) gives more information than (c).
√ We submit again the comparison between d e d + 1. The answer is quick:
d < d + 1
We state that obviously the writing is valid for the same score of the dice.

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Suitable age related activities	*6*	*7*	***8***	***9***	*10*	*11*	*12*	*13*	***Comments***

From the preceding situation comes the **Game of 'I Prefer'**

Diary 38 (third year, 8 year olds, May)

√ We propose the Game of 'I prefer':

On which card would you like your counter to be?

d	d + 1	d - 1

💬 The children show they understand clearly that it is better to be on d + 1 «because if we get a 6, then d is 6, and d plus 1 is 7 and d minus 1 is 5»
√ We propose an other situation:

On which card would you like your counter to be?

d – 1	d × 0

💬 «If we get 1 then d is 1, and 1 minus 1 is the same as d multiplied by 0»
√ We help the classes to build a chart with the possible values of the dice.

If d = 2 then $2 - 1 > 2 \times 0$
If d = 3 then $3 - 1 > 3 \times 0$
If d = 4 then $4 - 1 > 4 \times 0$

💬 A child's face lightens «A number multiplied by zero always gives zero!»
💬 We get to the collective writing:

If $d \neq 1$ then $d - 1 > d \times 0$

Expansion 2

The Game 'I Prefer' opens some interesting hypothesis for the following classes, as regards an approach to inequalities.
Some examples:

Example 1

On which of these two cards do I prefer my counter to be?

d+2	d+3-2

The comparison turns out to be simpler if I represent the content of the second card in a different way:

$$d + 3 - 2 = d + 1$$

The result of the comparison is now evident:

$$d + 2 > d + 1$$

It is convenient for me to have my counter on the first card.

Example 2

On which of these two cards do I prefer my counter to be?

2d	d + 4

This time it is better to represent the content of the first card in a different way: in form of addition rather than of multiplication:

$$2d = d + d$$

the comparison is now between the two writings:

$$d + d \quad \text{e} \quad d + 4$$

We explore the situation, by imagining the possible values of the dices and filling in a chart:

d	d + d	d + 4
1	1 + 1 = 2	1 + 4 = 5
2	2 + 2 = 4	2 + 4 = 6
3	3 + 3 = 6	3 + 4 = 7
4	4 + 4 = 8	4 + 4 = 8
5	5 + 5 = 10	5 + 4 = 9
6	6 + 6 = 12	6 + 4 = 10

We can sum up the chart like this:

$$2 < 2d < 12 \quad \text{and} \quad 5 < d + 4 < 10$$

The comparison brings now to a rich conclusion:
1) If $d < 3$ it's convenient to stay on the first card
2) If $d = 4$ being on the two cards is the same
3) If $d > 4$ it is convenient to stay on the second card.

(Continues in the following page)

Suitable age related activities	6	7	8	**9**	**10**	11	12	13	**Comments**

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Example 3

On which of these two cards do I prefer my counter to be?

d + 2	8 - d

Like before, we can write:

$3 < d + 2 < 9$ and $2 < 8 - d < 7$

This time the comparison is more complex. We'd better think: in which cases is it convenient (or is it indifferent) to be on the first card? We translate this question for Brioshi:

$8 - d \leq d + 2$?

This inequality can be solved by exploring the values of the two cards for each value of the dice:

	First card	Second card
d = 1	1 + 2	8 – 1
d = 2	2 + 2	8 – 2
d = 3	3 + 2	8 – 3
d = 4	4 + 2	8 – 4
d = 5	5 + 2	8 – 5
d = 6	6 + 2	8 – 6

To conclude: it is convenient if we get 3, 4, 5 or 6.

In advanced classes we can solve the inequality applying the principles discovered in Unit 6: 'From scales to equations':

$8 - d \leq d + 2$
$8 - d - 2 \leq d + \cancel{2} - \cancel{2}$ (1° principle)
$6 - d \leq d$
$6 - \cancel{d} + \cancel{d} \leq d + d$ (1° principle)
$6 \leq d - d$
$6 \leq 2d$
$6 : 2 \leq 2d : 2$ (2° principle)
$3 \leq d$

Suitable age related activities	*6*	*7*	*8*	***9***	***10***	*11*	*12*	*13*	***Comments***

Supplementary activity 1

(To be proposed also - if necessary - as an individual activity)

Problem 1

Alice has his counter on this coloured card:

Add 3 to the score of the dice

She substitutes the coloured card with the corresponding white one written in mathematical language.
In your opinion what's the writing on the white card?
Alice throws the dice and advances of 7 boxes.
I wonder what score she did, I didn't see the dice.
Can you help me? **35**

35

d + 3

d + 3 = 7
d + ~~3~~ – ~~3~~ = 7 – 3 d = 4

Problem 2

Marianna has her counter on this coloured card:

Subtract the score of the dice from 9

She substitutes the coloured card with the corresponding white one written in mathematical language.
In your opinion what is the writing on the white card?
Now Marianna throws the dice and advances of 3 boxes.
Can you guess the score Marianna got throwing the dice?**36**

36

9 – d

9 – d = 3
9 – ~~d~~ + ~~d~~ = 3 + d
9 = 3 + d
9 – 3 = ~~3~~ + d – ~~3~~
6 = d

Problem 3

Adriano has his counter on this coloured card:

Multiply by 2 two the score of the dice and subtract 2

He substitutes the coloured card with the corresponding white one written in mathematical language. In your opinion what is the writing on the white card?
Now Adriano throws the dice and advances of 2 boxes.
I wonder what was Adriano's score! Do you know it?**37**

37

2d – 2

2d – 2 = 2
2d – ~~2~~ + ~~2~~ = 2 + 2
2d = 4
2d : 2 = 4 : 2
d = 2

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Suitable age related activities	*6*	*7*	*8*	***9***	***10***	*11*	*12*	*13*	***Comments***

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Problem 4

Anna has her counter on this coloured card:

Double the score of the dice

Sandra has her counter on this coloured card:

Add 7 to the score of the dice and then subtract 5

Both Anna and Sandra throw the dice, and they move their counters.
Leo says: 'Wow, Anna and Sandra took the same number of steps! I wonder what score the two friends had got throwing the dice! Would you be able to find it? **38**

38

Anna: $d \times 2$ or $2d$
Sandra:
$d + 7 - 5$
$2d = d + 7 - 5$
$2d = d + 2$
$2d - d = d + 2$ or $d + \cancel{d} = \cancel{d} + 2$
$d = 2$

Problem 5

Mario is on this coloured card:

Halve the score of the dice

He throws the dice and obtains 5.
At this stage he does not what to do anymore? Why? **39**

39

5 : 2 He should move of two and a half boxes, and this is not possible

Problem 6

Pia is on this coloured card:

Add 1 to the score of the dice and halve

What values of the dice allow Pia to advance without any problems? **40**

40

The odd values (1, 3, 5).

Problem 7

Alice has her counter on this white card:

d + d

She throws the dice and moves of three boxes. What scores of the dice allow these advancement? **41**

41

Alice either made a mistake or she cheated. No double can be equal to 3.

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