A TASK AIMED AT LEADING TEACHERS TO PROMOTING A CONSTRUCTIVE EARLY ALGEBRA APPROACH

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We tackle the issue of how teachers can be led to acquire conceptions and models of behaviours suitable to foster among pupils a constructive and linguistic approach to the algebra since primary school. We propose a model of structured task, as a connected set of questions addressed to teachers and centred at the analysis of activities and pupils' productions as well as on the design of classroom discussions. This task is made of operative and critical reflection activities, aimed at favouring the development of predictive and interpretive thinking by teachers, with respect to pupils' behaviours. We briefly report on some results of its use in teacher training.

INTRODUCTION

The socio-constructivist approach to the learning of mathematics has some important implications for teaching. The first is that the teacher image raises to a higher dignity: (s)he is a person with an individual interpretation of reality, and in particular of his/her teaching discipline, and of the aims and tools of its teaching. The second implication is that mathematics teachers have the responsibility of creating an environment that allows pupils to build up a mathematical understanding, but they also have the responsibility to make hypotheses on the pupils' conceptual constructs and on possible didactical strategies, in order to possibly modify such constructs. This implies that teachers must not only acquire pedagogical content knowledge, in Shulman's sense (1986), but also knowledge of interactive and discursive patterns of teaching (Wood 1999).

The complexities of classroom and school life oblige teachers to continually make decisions. These decisions do not only involve the solution of problems arising in the classroom, but foremost of their identification. Lester & Wiliam (2002, p. 494) stressed that "*the speed with which decisions have to be made means that the knowledge brought into play by teachers in making decisions is largely implicit rather than explicit*". Thus, it is important that the teachers are able to recognize and control it. This implies that they must be able to analyse their actions and reflect on the reasons that produced them.

Some scholars points out the need for teachers to reflect on their own practices (Mason 1990, 1998; Jaworski 1998, 2003). Jaworski (1998, p. 7) uses the following words to define the kind of practice that results from such a reflection, i.e. reflective practice: "*The essence of reflective practice in teaching might be seen as the making explicit of teaching approaches and processes, so that they can become the objects of critical scrutiny.*" Through reflective practice, teachers become aware of what they are doing and why: awareness is therefore the product of the reflective process.

We consider awareness as an essential element in the construction of a teacher's

qualified professional identity, and agree with Mason (1998), who emphasizes that what supports effective teaching is “awareness-in-counsel”. Thus it is extremely important that teachers undergo some preliminary training about the dynamics involved in the teaching and learning processes and particularly about aspects that influence decisional processes. The core objective in these teacher training processes is to make teachers aware that pupils are the main builders of their own knowledge.

In this perspective, on one hand teachers need to be offered chances, through both individual study and suitable experimental activities, to revise their knowledge and beliefs about the discipline and its teaching; on the other hand, they need to become aware that their main task is to make students able to give sense and substance to their experience and construct new knowledge by exploring situations and making links with familiar concepts and objects.

The actual attainment of these goals is extremely complex in the case of classical thematic areas, such as arithmetic and algebra, which suffer from their antiquity, and teaching of which is affected by the way they historically developed.

Teacher training referring to *early algebra* currently raises a great research interest: this disciplinary area became increasingly important in the last decades, as an answer to issues related to the teaching and learning of algebra. The work we present here concerns this content area and deals with activities and methods to educate teachers, aiming at a renewal of its teaching in a socio-constructive perspective.

WHY EARLY ALGEBRA

One of the most heartfelt problems for secondary school teachers concerns difficulties students meet in their approach to algebra. The main reasons of these difficulties essentially lie in the heavy loss of meaning felt by students about the objects of study.

Since the ‘80s, research pointed to a way to modify this situation, underlying the need to promote since primary school a pre-algebraic teaching of arithmetic, cast toward the observation of numerical regularities, the recognition of analogies, generalisation and an early use of letters to represent observed facts.

Starting from the second half of the ‘90s, many theoretical and experimental studies were carried out on these aspects, mainly addressing 11-13 year-olds. Some of these studies stand out due to a theorisation of socio-constructive models of conceptual development; they emphasise the influence of the class environment on learning and promote the use of physical tools as means of semeiotic mediation, in the frame of a view of algebra as a language (Da Rocha Falcao, 1995; Meira, 1990, Radford 2000).

Since 2000 broad studies concerning teaching experiments, carried out at primary school level in relation to algebraic contents, appear (see for instance Carraher & al., 2000 or Carpenter & Franke 2001); some of these studies also concern the setting up of projects aimed at training primary school teachers on these issues (Blanton & Kaput, 2001; Brown & Coles, 1999; Dauguerty 2001; Menzel, 2001). Our *ArAl Project*, *paths in arithmetic to favour pre-algebraic thinking* locates within this frame (Malara & Navarra,

2003): it is a project that merges teacher training and innovation in the classroom.

THE ALGEBRAIC BABBLING

In the traditional teaching and learning of algebraic language the study of rules is generally privileged, as if formal manipulation could precede the understanding of meanings. The general tendency is to teach the syntax of algebra and leave its semantics behind. We believe that the mental framework of algebraic thought should be built right from the earliest years of primary school – when the child starts to approach arithmetic – by teaching him or her *to think of arithmetic in algebraic terms*. In other words, constructing algebraic thought in the pupil progressively and as a tool and object of thought, working in parallel with arithmetic. It means starting with its *meanings,* through the construction of an environment which might informally stimulate the autonomous processing of that we call *algebraic babbling*. We hypothesize that there is an analogy between ways of learning natural language and ways of learning algebraic language. The *babbling metaphor* can be useful to clarify this point of view: while learning a language, the child gradually appropriates its meanings and rules, developing them through imitation and adjustments up to school age when he will learn to read and reflect on grammatical and syntactical aspects of language.

The perspective to start off the students with *algebra as a language*, continually thinking back and forth from algebra to arithmetic, is based on the negotiation and then on the rendering explicit of a didactical contract, in order to find the solutions of problems, based on the principle "*first represent, then solve*". This perspective seems very promising when facing one of the most important issues in the field of conceptual algebra: the transposition in terms of representation from the verbal language, in which problems are formulated or described, to the formal algebraic language, into which relationships are translated. In this way, the search for the solution is part of the subsequent phase. From this point of view, *translating* sentences from verbal (or iconic) language into mathematical language, and vice versa, represents one of the most fertile areas within which reflections on the language of mathematics may be developed, even for the deep differences between the morphologies of the two languages. "Translating" in this sense means *interpreting* and *representing* a problem situation through a formalised language or, conversely, recognising a situation described in symbolic form.

Such an innovative vision requires a process of authentic reconstruction of teachers' conceptions in the field of mathematics and methodology, which is also among the objectives of the project itself.

Methodological aspects in the approach to early algebra

The proposed situations in our approach to early algebra are developed within stimulating teaching and learning environments, but they are not easily manageable by teachers and involve several delicate aspects requiring specific competencies. As a consequence, those who wish to undertake innovative educational practices, need to deal with their own knowledge and beliefs, but also with a set of relevant methodological and

organisational aspects that actively support a culture of change. We shall now discuss some of these aspects.

The didactical contract

A continuous check of the clarity of the didactical contract implies that one constructs in pupils mathematical conceptions that will possibly favour a gradual formation of algebraic thinking, rather than give them technical skills. However, pupils must be aware that the essence of the contract is that they are protagonist in the collective construction of algebraic babbling. This means they should be educated to gradually become sensitive towards complex forms of a new language, through a reflection on differences between and equivalences of meanings of mathematical written expressions, a gradual discovery of the use of letters instead of numbers, an understanding of the different meanings of the 'equal sign', the infinite representations of a number, the meaningful identification of arithmetic properties and so forth. When the pupils realise that they are producing mathematical thinking and contributing to a collective construction of knowledge and languages, they make a variety of proposals, mostly interesting and non-trivial that altogether represent a common legacy for the whole class. It is here that the teacher should favour identification by pupils' paraphrases of a possibly correct translation, by selecting wrong, ambiguous, redundant, misleading or fancy translations.

Mathematical Discussion and teacher' s role

The enactment of a collective discussion on mathematical themes leads to privilege metacognitive and metalinguistic aspects; pupils are guided to reflect on languages, knowledge and processes (like solving a problem, analysing a procedure), to relate to classmates' hypotheses and proposals, to compare and classify translations, evaluate their own beliefs, make motivated choices. In all this, the teacher should be aware of the risks and peculiarities of this teaching and learning mode.

The teacher plays a delicate role in orchestrating discussions. First, he/she must be clear about the constructive path along which pupils should be guided, and about the cognitive or psychological difficulties they might encounter. From a methodological point of view he/she must try to enact the various voices in the class harmoniously, inviting usually silent pupils to intervene, avoiding that leaders and their followers prevail and that rivalries between groups come out. Finally, he must help the class recognise what has been achieved as a result of a collective work involving everybody. He/she must learn to act as a participant-observer, i.e. to keep his/her own decisions under control during the discussion, trying to be neutral and proposing hypotheses, reasoning paths and deductions produced by either individuals or small groups. He/she must learn to predict pupils' reactions to the proposed situations and capture significant unpredicted interventions to open up new perspectives in the development of the ongoing construction. But this is a hard-to-achieve baggage of skills.

THE PRESENT STUDY AND ITS METHODOLOGY

In these last years we addressed our research towards the individuation of methodologies and tools aimed at producing in teachers the mathematical/pedagogical competencies necessary to approach early algebra in a socio-constructivistic way.

Our research methodology is based on: 1) planning of didactical classroom routes with the teachers; 2) teachers' production of *diaries* (i.e. teachers' transcripts of audio-recordings of the classroom activities intertwined with their local or general comments and reflections); 3) joint (researcher & teacher) analysis of diaries; 4) constant sharing of the diaries among the teachers involved, writing of meta-comments, discussions and reflections.

Our research experience with the teachers made us aware of the difficulties they meet as to the design and management of whole class discussions (Malara 2003). This convinced us about the importance of the classroom processes analysis in order to lead the teachers to acquire the necessary competencies for the orchestration of the classroom discussion (activation of provisional thinking, listening and coordination of the students' voices, ability to take prompt decisions, ability to improvise, etc).

The collected documentation on the classroom episodes analysis - paradigmatic in highlighting the sharp correlation between the students' mathematical constructions and the teacher's sensitiveness in the discussion, - brought us to elaborate a model of task we focus on.

A MODEL OF TASK FOR TEACHERS: THE ANALYSIS OF CLASSROOM SCENES IN SEQUENCE

Our model of task gives teachers the chance to deal with the practice of constructive teaching 'theoretically', forcing them to focus on provisional and reflection-related aspects. The core of this task is to lead teachers to be able to analyse specific class processes and *reflect* upon these processes at different levels (development of pupils' mathematical construction, teacher's action, individual participation etc). The task develops along *Some Scenes (5 o 6*), structured as a 'connected set of issues', and a *Final Reflection.* The scenes are based on excerpts of transcripts of one of our experimentations. Each *Scene* is a partially autonomous activity and 'realistically' sets the teachers in a class situation. It is composed of two sections: the first concerns the presentation of a classroom situation diary, and the second centres around questions related to the synthesis, in which teachers are asked: to tell their opinions about how meaningful the task proposed to pupils was; or to make hypotheses on pupils' productions and difficulties; or to interpret individual productions; or produce sketches of discussions aimed at comparing pupils' productions. As a whole, the set of Scenes put the teacher in front of a simulated classroom process.

Teachers are sequentially proposed Scenes at regular time intervals (on average 20-30 minutes): while they are working on the first Scene they do not know the second yet; when they elaborate on the second one they do not know the third one yet and so forth

up to the conclusion. After analysing the input proposed in the first part of the Scene, the teacher makes hypotheses about the class' reaction. In the subsequent Scene these hypotheses are compared with what actually happened and so forth up to the last Scene. At the end a global review of the work done is proposed, in order to reflect on: the process analysed from the point of view of mathematical construction and educational value for pupils; how meaningful the task is in terms of professional development.
This kind of task developed with time and can be seen as the result of a research process. The first idea was born on producing interconnected e-learning worksheets for teachers in order to promote a constructive and linguistic approach to algebra, to which a transposition of these materials in a first version of the task for teacher workshops followed. The present version of the task has been proposed in workshops for teachers/trainees of junior secondary school since 2005. [1]

AN EXAMPLE OF TASK IN EARLY ALGEBRA

We present a prototype of task made of six scenes (selected by a didactical process experimented in a 6th grade) and a final reflection. The process develops from an individual pupils' activity concerning the individuation of the rules and their algebraic formulation (see the first scene). This task was given to 45 teachers as the final test of a weekly 40-hour course on didactics of algebra, where didactical and methodological questions on early approach to algebra were faced with reference to the most recent research. Here, 20 hours were devoted to workshops dealing with similar tasks together with the analysis of excerpts from class discussions. The aim was to test the effectiveness of such task as an element of mediation between theory and practice and, more specifically, as a tool to set traenees 'inside' a virtual class, so that they can get aware of what it means to 'act in the moment'.

The first Scene

The class teacher presents the situation:
Tilde likes chocolate cookies and finger biscuits, which she eats for breakfast every school day. She eats different quantities every day, but follows a rule she set.
Then she shows a drawing with two ice cream spots which only hide the finger biscuits Tilde ate on Friday and the chocolate cookies she ate on Saturday.

Monday	Tuesday	Wednesda	Friday	Thursday	Saturday

At this moment the teacher gives a first task: Write down Tilde's rule in natural language.[2]

Task for teachers: 1. Carry out the task. 2. Give a short explanation about why searching for regularities is important in mathematics. 3. Discuss the instruments and/or strategies you view as the most efficacious to identify regularities.

The second Scene

Pupils' sentences are classified by the class teacher and the most representative ones are written on the blackboard. Then the teacher opens up the discussion with the purpose of making the class decide what sentence best represents the rule followed by Tilde.

> *(a) Tilde takes from finger biscuits the same number of chocolate cookies, she multiplies it by 2 and adds 1.*
> *(b) She eats an odd number of finger biscuits and from 1 to 5 chocolate cookies.*
> *(c) Chocolate cookies are always one more.*
> *(d) One day she eats more and one day she eats less.*

Task for teachers: 4. Comment upon each of the four sentences written on the board concerning their consistency, completeness and efficacy with respect to the formulation of the regularity. 5. Imagine a plot for a discussion: opening, key steps and end.

The third Scene

After a collective discussion, pupils agree on the choice of (a):

> *(a) Tilde takes from finger biscuits the same number of chocolate cookies, she multiplies it by 2 and adds 1.*

The class teacher proposes 'Each of you may try to write (a) in other ways'
After a while some significant expressions written by pupils are reported on the blackboard:

> *(a_1) The number of finger biscuits is 1 more than twice the number of chocolate cookies.*
> *(a_2) The number of finger biscuits is twice plus 1 the number of chocolate cookies.*
> *(a_3) The number of chocolate cookies multiplied by 2, adding 1 equals the number of finger biscuits.*

Based on these sentences the teacher opens up a new discussion.

Task for teachers: 6. Comment upon the choice made by the class to send sentence (a). 7. Analyse the last three sentences, highlighting any possible difference with respect to the relational-procedural polarity. 8. Figure out a plot for a possible discussion.

The fourth Scene

At the end of the discussion the class chooses sentence (a_1); the remaining two are erased.

> *(a_1) The number of finger biscuits is 1 more than twice the number of chocolate cookies.*

The teacher gives a second task: (2) Translate Tilde's law for Brioshi in algebraic language [3]

Task for teachers: 9. Translate Tilde's law in more than one way. 10. Predict the possible translations by pupils.

The fifth Scene

After an individual work, sentences are written on the blackboard (pupils'comments in brackets):

1.	1 × 2	
2	a + 1 × 2	(a = number of finger biscuits)
3	fb + 1 × 2	
4	a × 2 + 1	
5	fb + 1 × 2 = a	
6	fb = ch + 1 × 2	
7	a = b × 2 + 1	
8	a × 2 + 1 = b	(a = number of chocolate cookies)
9	(a – 1) × 2	

A discussion is enacted to choose the sentence that should be sent to Brioshi.

Task for teachers: 10. Comment upon each translation of Tilde's law, underlying their correctness/ incorrectness, consistency, possible redundancies etc. 11. Predict what the class will possibly choose and related argumentation about the sentence they will send to Brioshi.

The sixth Scene

At the end of the discussion about the sentence to be sent to Brioshi, the following rule is chosen: b = a x 2 + 1. The class teacher poses the problem •*Are we able to understand what happens on Friday and Saturday although ice cream spots hide part of the drawings?'* Some pupils almost immediately use the formula correctly but other pupils are initially puzzled. This difficulty is overcome during discussion.

Task for teachers: 12. Is the rule sent out by the class the same you predicted at the end of the Fifth Scene? Is it different? Write down your comments on this. 13.Identify what the spots hide in the table illustrating the biscuits Tilde eats. 14. With relation to the question posed by the teacher, interpret the reasons underlying the widespread confusion in the class.

Final reflections

15. write down a short reflection on the didactical situation you were asked to comment upon, also referring to a possibility of reproducing it in a hypothetical class of yours.
16. Write down a short concluding reflection on the structure of the whole set of tasks, mainly referring to its significance as model of task that may help trainee teachers explore what they learned both at mathematical and pedagogical level.

We cannot refer about the teachers' behaviour (we shall give some specific indications in the WG), so we limit ourselves to indicating some of the main results concerning the last questions of the task: 'the final reflections'.

ABOUT THE TEACHERS' REFLECTIONS

Teachers' reflections gather around two aspects, often interwoven: value of the task as guiding instrument for the teacher; complexity of the task, difficulties and fears this generated.

The protocols show the undoubted usefulness of the task as a valid instrument to lead teachers, and especially those who have a little class teaching experience, to approach issues that characterise the teaching activity such as: a) prediction of pupils' behaviours;

b) interpretation of their productions; c) setting up of discussions; d) sensitivity in grasping not only potentialities of pupils' contributions, but also obstacles produced by their distorted or partial views, often difficult to be identified. The structure of the task, divided into scenes, is considered an effective classroom simulation to guide the teacher in a step-by-step involvement with the teaching sequence. Hypotheses about classroom-based actions are seen as very demanding tasks, still it is highlighted that the task is a powerful mean to force the teacher to compare predictions with the actual development of the class activity.

Protocols also show teachers' difficulties and fears concerning their own future abilities to implement constructive teaching. Difficulties mainly concern the design of discussion plots, the interpretation of and comment on formal written expressions, identification with students. Other fears concern not being able to understand pupils, confusing them, not being able to make them actually discuss, implementing a directive way of piloting the class, due to an unconscious stereotyped model of teacher.

Moreover, the protocols underline that teachers are aware of the cultural importance of expliciting their own development as teachers and open to collective sharing.

The processes in which they share new knowledge and new meanings are enacted at two different levels: with oneself, leading to reflections on the course's and the task's impact on one's own professional development and, more generally, on the 'renewed' relationship between theory and practice; with colleagues: by means of a peer to peer comparison about the task results and the things learned in the course, this leads teachers to become aware of the features of their own beliefs as well as of their way of conceiving the classroom activity. The latter is a fundamental step for a review of one's own attitudes and a change of unsuitable ones.

NOTES

1 In order to become a secondary school teacher in Italy, one has to achieve a diploma from a two-year 'Scuola di Specializzazione all'insegnamento' (Specialization school for teaching) after graduating. But in Italy it is frequent that graduates are engaged as temporary teachers before and during their attendance at the Specialization School for teaching. For this reason we write here 'teachers/trainees' but in the following we shall simply write 'teachers'.

2 The question posed to the pupils is not easy: the text contains subtle logical elements, involving the meaning of both the universal quantifier 'all' and the adverb 'only', and uses spots to pose the problem of identifying unknown data. By examining the drawing, pupils must find a link between the number of finger biscuits and the number of chocolate cookies they see in the six boxes, trying to identify the relationship linking them. This is not a difficult task if one tries to express the number of finger biscuits as a function of that of chocolate cookies; much more difficult is to express the relationship starting from the number of chocolate cookies. Other two interrelated aspects, that should not be underestimated, concern the issues of the interval in which the single quantities may vary and generalisation of the law (the situation might induce a cyclic view of the process and inhibit a general view of the law). Moreover the Saturday day case arises some psychological and mathematical questions as to the number 0 for the lack of chocolate cookies under the spot.

3 In order to help pupils get to know the problem of the algebraic representation of verbally expressed

relations or procedures, we invented a metaphorical character called Brioshi, who turns out to be a very effective support on conveying pupils aged 8-14 that it is necessary to respect the rules in the use of a symbolic language.

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