ITINERARIES THROUGH LOGICS FOR POTENTIATING LINGUISTIC AND ARGUMENTATIVE SKILLS.

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PREFACE

One of the main problems which has to be faced daily by teachers is *the difficulty the students find in oral and written expression*, and it is well known that *linguistic competence* is an essential condition for learning quite apart from the disciplinary context, even the more technical ones, which apparently require almost exclusively instrumental skills.

A complex aspect of linguistic competence lies in the ability of organizing one's opinion in an exhaustive and communicable way; in other words, in the ability of *explaining the stream of a reasoning* which starting from one or more premises comes to a conclusion[2].

In order to face these items, we have been leading since '86 (constantly attending to the lessons together[3]) in collaboration with the teacher of Arts in the same course Laura Buzzatti, aimed at potentiating the linguistic skills in students aged 11 to 14. It is a teaching activity within Logic, seen particularly as a *transversal area* - in comparison with linguistic and scientific-mathematical areas - and as a *reflection on language*.

After a short overview on the main objectives of our work, we shall analyse three examples: To argue and to explain (11 years), Not banal reflection activities (12-13) and Deductions of a shrewd inspector (13-14) through which we shall illustrate the strategies followed and comment some results through the analysis of yhe students' answers.

AIMS OF THE ACTIVITY AND METHOD OF WORK

The main aims are:

1) the improvement of linguistic skills through:
   - the development, by means of discussion, of *critical thought*, that is the capacity of assessing the pertinence, the errors, the potentialities of a speech;
   - the order in organising and communicating ideas respecting priorities, coherence among concepts, property, etc.
   - the capacity of recognising communicative and metalinguistic levels.

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[2] To do that, it is necessary for the pupils to be *conscious* of their knowledge - apart from its scholastic or extra-scholastic origin - and to be able to use it in a flexible way. It would be important to ascertain how much the training and the organization of the pupil's knowledge can have an influence - positive or negative -on his skill in making internal connections. In other words: till which point is it possible for him to reconstruct his knowledge in front of new problematic situations if it has not already been constructed from the beginning in order to favour the flexibility?

[3] . The Italian law allows this kind of cooperation within a particular arrangement of the school time called “tempo prolungato”.

2) The comprehension on the one hand of the *richness* connected with the ambiguity of the natural language and, on the other hand, of the *power* of a formalized language. In other words:
   • the singling out of the differences between semantic and logical-syntactical aspects of a talk;
   • the conquest of the concept of *symbol*;
   • the awareness of the relation between symbols and rules which link them;
   • competence in using instruments like truth tables.

Some of the more interesting aspects of this activity concern the active role of the students in the direction of discussions. In fact, often, original observations or points of view bring them to directions not only unexpected by the teachers, but also highly *sophisticated* on a conceptual level for the students of this age. Some other times a debate is produced between the two teachers in which the students take part in a constructive way, as *protagonists*, and the continuous interferences between *semantic* and *logical* aspects create the premises to very interesting reflections.

We use strategies which need different instruments: graphic instruments (Eulero Venn's sets, Carrol's square, graphic trees), iconic instruments (transparencies, drawings, slides), manipulatory instruments (Vygotsky-Dienes logical blocks, magnetic letters, playing cards, etc) but linguistic instruments are preferred: texts and tales invented by the teachers, which can be considered *logical problems* (Malara, 1989 and Navarra, 1992, 1993).

The large number of texts for every subject treated (connectives, quantifiers, 'modus ponens', and other reasoning figures) proposed in sequence of growing linguistic complexity and logical difficulty, the flexibility in the running of sequences, the accurate diagnosis of arguments and linguistic difficulties, the attention paid to the interaction student/student promote a progressive refinement of the critical skills of the students in assessing and comparing the adequacy of their own arguments and those of others.

The structure of this course allows the teachers to find out difficulties and abilities which, whilst for some aspects can be well known to the teachers, and confirm their knowledge of the students, from other points of view can expand it bringing the observation towards usually less practiced fields by the traditional teaching, like cross-comparison, comment and correction of the answers, with the aim of analysing the validity of some and the linguistic clearness used to express them.

Such a standard of discussion can be associated with the *balance discussion,* carefully experimented and examined by other researchers (see, for example, Bartolini Bussi, 1991); of course we don't ask the class to examine *all* the answers, but just a *selection* of them, those which represent more clearly the different strategies, and in this context the discussion is planned, constantly carried out with the essential support of the over head-projector, used to project the answers the students have transferred onto transparencies.

We usually verbalize the lessons in order to retain the richness of what is said during collective discussions, which otherwise would not be easily grasped. This enables to reflect more carefully on the lessons already developed, and to plan the next lessons with more elements of judgement, which is undoubtedly simplified by the presence of both teachers in the class, which enables them to play, according to the different situations, the role of the *animator* and of the *reporter.*

ANALYSIS AND COMMENTARY ON SOME MEANINGFUL STAGES

A. TO ARGUE AND TO EXPLAIN (11 year old students)

In front of problematic situations of high logical content, very often the students elaborate personal strategies in order to overcome cognitive hindrances, and they change the situations given into others - not necessarily coherent with those given at the beginning - in which they think they are enacting a higher solving ability. It is of great interest for a teacher to examine these performances and to try and identify the table of logical reference that has been produced by the students.

Let's analyse as an example a starting experience conducted with eleven year old students, before beginning the real didactic-logical activity.

The text proposed is an adaptation of Smullyan's famous logical problem about *Knights and Knaves* (Smullyan, 1978). The protagonists are a survivor from a wreck and the inhabitants of an island where the *Knights* always tell the truth and the *Knaves* always tell lies. The students, playing the part of the survivor, meet people; because they do not know *a priori* who is a Knight and who is not - they have to discover the identity of the various characters according to the answers they give to their questions.

> Here comes the first inhabitant of the island. You look at him, but apparently he could be either a knight or a knave. So, without thinking, you ask: «Who are you, a knight or a knave?»
>
> What has the inhabitant of the island answered?

Let's examine the prototype answers:

> Rosella: “He answered: ‘I am a knight’, but he knew he was not. He said so because he wanted the knaves to become more numerous than the knights, therefore he knew or imagined that the knaves could become stronger than the knights”.
>
> Christian: “I think he says he is a knight because if he is a knave he says he is a knight and if he is a knight he says he is a knight. My opinion is that the survivor should be able to understand it because if the inhabitant says he is a knave, he is a knight, and if he says he is a knight, he is a knight.”
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> Elisa: “ He says ‘I am a knight’. I think he says so, because if he is a knight, he tells the truth and he introduces himself as a knight, and if he is a knave he tells a lie and, instead of introducing himself as a knave, he presents himself as a knight.”

It is clear that Rosella has given an imaginative answer which can be assimilated into an *argument*, meaning that “...every argument is personal, and addresses itself to people whose assent it tries to gain...” (Perelman, 1977, p.791). Somehow, Rosella is trying to convince the audience, following *her logic* which is referring to situations and elaborations which are completely *personal*.

Elisa's answer on the other hand looks somehow like a *demonstration*, although *naive*, as she deduces “...with absolute certainty conclusions starting from premises, given according to rules previously formulated...” (Perelman, 1977, p.791).

Christian gives an answer similar in the structure to Elisa's one, but he has not a complete logical control, and the second part (wrong) contradicts the first (correct).

In this class, just three students give answers like *Elisa-model;* to various extents the other sixteen follow *Rosella-model.* Almost everyone, however, while discussing the texts, set in front of the two models, understands that only Elisa's answer convinces completely, while the other ones can be funny, strange, peculiar, but they are less *sustainable.* Also, they notice that the explanations following *Elisa-model* are similar, because *they show evident analogies in the way the reasonings are structured*; the others, on the contrary, are different from each other, and can never be practically compared to one another. It's a difference similar to the one between *hypotheses* and *opinions* elaborated by the students during the construction of simple mathematical models, and also studied by the Research Group of Genova , Italy (Boero, 1990).

Because such a situation emerged also in other occasions, we can say that generally there are few students who can recognize spontaneously *the deductive connections* among the elements of a problem and expound them in a coherent form. As far as the others are concerned, many of them, even when put in front of an individual activity, without the contribution of the collective discussion, don't improve their performances in a significant way. It is anyway important for them to take part in the construction of the *test* of a reasoning, and to grasp the processes through which the construction is realized. In our opinion their participation becomes much more significant if we consider it within the Vygotskijan theory of zone of proximal development (Vygotskij, 1978).

Often, especially with linguistically weaker students (but not only with them), wrong knowledge is confused because of the lack of ability to control the meaning of very common words that the teachers consider *as known.*

A relevant example has happened with 12 year old students during a test stage on paraphrases of the connective ‘and’: also/too, on the contrary, however, but, etc., with the aim of leading the students to the identification of a *logical characteristic* common to words which have on a semantic level very different shades of meaning.

In comparing the possibility of a logical equivalence between two sentences:

> «I assure you» Aldo is saying «the trout is a fish, *on the contrary* the dolphin is a mammalian».
>
> and
>
> «Absolutely no!» Cosimo answers back «the trout is a fish, *however* the dolphin is a mammalian!»

A student below the everage openly declared his difficulties of interpretation: “Cosimo's answer could be right, but I cannot tell because I don't understand the difference between *however* and *on the contrary*. It is not an easy task for the teachers to value answers of this sort once they come out, and to choose the best strategies to face the difficulties they point out. This happens also because many times a confused explanation of the student brings to the discovery of the existence of the problem, but not so clearly as to plan an appropriate therapy, especially with students who have already experienced six or seven years of school, often predominantly frustrating.

Certainly, however, logical linguistic activities as these proposed and the usual reflection particularly to the verbalisation (oral and written) as constant didactic procedures can represent a very favourable *pedagogical environment* in this sense.

B. NOT BANAL REFLECTION ACTIVITIES (12-13 year old students)

One of the most meaningful aspects that emerges in the course of our work is the increase of the *metacognitive level* the students are gradually using, because they are constantly obliged *to reflect on their reasonings and on those explained by the schoolmates* (see Schoenfeld, 1985). The purpose we would like to fulfill is, besides the developement of the linguistic abilities, also the refining of their capacity to find a *self direction of intellectual work* (Boero, 1990), in order also to improve their competence in the field of mathematics as, for example, those necessary to consciously face problematic situations.

Here is reported, as an example of these last observations, a *diary* of the discussions treated in the classroom, concerning this text:

Greedy girls

> Eloisa and Giovanna are admiring the window of the confectioner 'Super-cake'.
> «Mmm, come on, let's go in!» says Eloisa «first of all *I'm going to have a strawberry pastry and a chocolate shake!*»
> «I can't see how you will do it with the little money we have: *surely you can have the pastry ..... have/ing the shake!*» Giovanna observes searching in her pockets.
>
> Place in the blank the connective that, in your opinion, Giovanna has used.

After a starting stage of individual work the discussion produces the following suggestions:

(a)	but not	8 students
(b)	or	6 "
(c)	without	1 student
(d)	maybe	1 "
(e)	and	1 "
(f)	or else	1 "
(g)	and then	1 "

The discussion brings the students to very interesting conclusions on the level of their comprehension of the meaning of the connectives.

Some of them observe that (e) and (g) are wrong because they contradict the context, in which Giovanna's perplexity concerning the scarsity of money is evident : it is so little that it's not enough for Eloisa's hypothetical bellyful, so it won't be possible "to have pastries **and then** the shake".

(d) 'It could be possible' suggests someone 'with dots' ("**and...maybe** to have a shake") but it is still wrong for the same reason as (e) and (g): Giovanna is sure of what she is saying.

And now for the most meaningful contrast: (a: “you can have the pastry **but not** the shake!”) and (c: “you can have the pastry **without** the shake!”) are wrong because - according to the students - *they create a hierarchy of preferences*; (b) and (f: “you can have the pastry **or else** the shake!”) on the contrary are right because *they do not change the meaning of the sentence*, that means, because it is impossible to know Eloisa's tastes, it is not possible then to state whether she would prefer the pastry or the shake, as on the contrary do (a) and (c) (there is the *hierarchy!)*. In conclusion, (a) and (c) **do change** the meaning of the text, (b) and (f) **do not change** it.

It is clear that during these reflections the contribution given by the students is very differentiated; certainly however, also because of the strategy of constantly writing the students' observations and suggestions on the transparencies, the whole class is busy in a very significant performance and highly refined at a conceptual level (comparisons, cross corrections, changes of mind, self-criticism etc.) in which the knowledge in question is run both by the teachers and by the students.

C. DEDUCTIONS OF A SHREWD INSPECTOR (13-14 year old students)

Between the age of 11 and 14 the capacity to work in competence of growing abstraction matures in the students; the teachers are offered the possibility, for what concerns logical-linguistic situations, to start some activities which involve, even though at an early stage, higher symbolical and formal aspects. It is possible to replace reasonings freely directed by the students with others which imply a more structured knowledge of some subjects as, for example, *reasoning figures* and the research of their validity.

In the situation we are going to examine, one of the aims is to compare in the class the *spontaneous methods* with those of *deductive logic*, in order to improve the organisation and explanation of the students' thought and their capacity to argue and to favour the transition from a confused or indifferentiated analysis to a more clear and coherent one.

We decide to invent some thrillers, and we found a *School for detective-students* in order to help Inspector Clouseau to link logically the clues and to draw the conclusion, in other words the solution of the case.

This is a story proposed in an examination test at the end of this activity, in which the students must spot out the guilty's characteristics by using strategies and *rules* (for instance: Clouseau *is always honest)* previously deepened.

The adventures of inspector Clouseau on the Nile

Right at the end of the adventure described in ‘Death under the pyramids’ the boat began the navigation again. A few days of calm have passed, when one night the inspector is suddenly awoken by the captain of the boat «Inspector! Wake up! Someone has killed lawyer Fitzgerald! It's terrible!»

The poor inspector must interrupt his sleep and set to work again. As usual the inquiries are complicated, and the end of the day Clouseau is reflecting on the deposition he has heard.

> «Well... the guilty person could be either one of Fitzgerald's collegues or one of the German tourists, I'm not certain yet... but of one thing I'm sure: the witness who told me that the killer is a lawyer and a short person, is a liar! And I've understood that even mr. Ross, who assured me that the killer is not American, lies... then, let's see: considering that...»
>
> At this point of the inquiries, the suspicions of inspector Clouseau are still on all the tourists, nobody excluded.

Before examining the results of the test, let's analyse briefly the difficulties the students must overcome:

1) to identify the three circumstantial evidences:
   (a) "The guilty could be either one of Fitzgerald's collegues or one of the German tourists."
   (b) "The witness who told me that the killer is a lawyer and a short person, is a liar!"
   (c) "Mr.Ross, who assured me that the killer is not American, lies."

2) to explain by writing in a clear way the connections between the evidences so as to distinguish the killer from the guests of the boat. To do that it is necessary to identify the *logical* aspect of the three evidences, built around the presence of connectives and truth values:
   (a) the first evidence contains an inclusive disjunction (either...or); its general truth value is *true* (the inspector is sure of what he's saying, and we can trust him; as we have said before, Clouseau is always *honest*).
   (b) The second evidence contains a conjunction (and) and its truth value is *false* (the witness is a liar!).
   (c) The third evidence contains just one statement and it's truth value is *false* (Mr.Ross lies).

Referring to the truth tables and selecting the possibilities compatible with the relative truth values of the periods, the students can recreate Clouseau's reasoning.

According to the quality of the explanations (relative to this text and to other logical problems) we can divide the students in three groups (relative to a pentenary distribution A, B, C, D, E). It seems possible to draw the partial conclusions that clearness in the control of the logical contents and clearness in the linguistic expression are strictly linked to one another.

A/B group

The students show their higher linguistic competence carrying out the analysis of the stories correctly and understanding clearly their logical contents. Also the synthesis is generally arranged in a coherent way (sometimes it may be too hurried), through a tidy selection of the sentences which allow the conclusion to be reached.

The logical mistakes are of little importance and anyway easy to find, and the students are able to correct them consciously.

Most of the students organize a discursive answer, sometimes too complex, even prolix, to the detriment of clearness and in other cases more schematic; someone make, even, an improper *stenographic* use of logical symbols, and the teachers of maths know very well that this is, generally, a frequent stereotype while working with algebraic symbols.

C group

The students commit inaccuracies on various levels of the analysis, interpreting in a partially incorrect way the connectives or the truth indicators or going hastily and superficially to the conclusion, even when correct in its substance, without clearly articuling the synthesis.

Mistakes cannot always be easily found (sometimes they are concentrated, in other cases sprinkled in the course of the reasoning, and the teachers have to face partly different strategies and a long time for reducing the causes that have induced them.

The text is often inaccurate and it is prevalently composed of schematic parts, linked by short discursive parts; the whole work points out a general linguistic poverty.

The logical and linguistic levels are in part confused; the students cannot overcome a kind of *detachment* by the logical sense of what they are doing, to which they remain in one sense *outsiders.*

To take advantage of the connections between the situations they are examining *they need the others,* to listen to their arguments, and to discover in them *a posteriori.* the clearness they have in an incomplete way. Often they acquire a reasonable control of the instruments, but without *the sense of the context* in which to use them.

D/E group

These students show misconceptions related both to the identification and to the interpretation of the logical contents of the text. The reasonings are linked again to a *personal logic* (the *Rosella-model,* in spite of the two years which have gone by since the activity about Knights and Knaves) and they are built tentatively. The students concentrate their efforts only on the analysis and interrupt it before having drawn some conclusion, or get to a wrong conclusion in an incoherent way .

The mistakes are many and diffused and the strategies to correct them, at least in part, are more complex to define and require individualized interventions.

There are no *series of problems:* every problem appears as a special case, so it must be faced with improvised instruments and strategies. The students are hardly aware either of their knowledge or of the linguistic means proper to express it.

The differences among many D/E-students' texts and the others are evident: they seem worried above all they won't be able to *convince* the teachers, and for this reason they use in a confusing, redundant and stereotyped way every kind of language they have learned during the activity, making a lot of mistakes. Evidently the didactic agreement [3] is not clear to them.

A-B students, on the contrary, are able to select the information, concentrating only on the necessary ones, and creating with them synthetic texts.

The contrastive analysis of texts so different to one another and then the explicit research of the reasons for which one text is objectively more clear than another, the need of *taking a stand* in the critical assessment of the texts are situations which have a high didactic value and allow the students to spot out some of the main reasons of the clearness of a reasoning: the information correctly used, neither superabundant nor poor reflections and explanations, the parts of the reasoning positioned in a very comprehensible order (from the premises to the conclusion), more explanatory clearness, no confusion among different languages, the syntax, the ortography and the grammar correct, and, on the whole, a higher formal order.

Moreover, two *styles of work* come out: to anticipate immediately the conclusion (chosen by the majority), or to put it at the end of the reasoning; one method is not necessarily better than the other. During a propaedeutic work it is maybe more advisable

to follow the second way, that obliges a more *methodical* work (*first* I analyze the data, and *then* I can draw the conclusions), also to avoid the *mania of the result* of many students which, at least at the beginning, could be mitigated by the duty to follow more strictly a plan of work.

As we have previously underlined, it is not easy for the weaker students to transform the results of the discussion in personal competence, and this happens particularly when they are not used to explain and to argue. On this point, it must be admitted that often there is no homogeneity among the teachers - even of mathematics (see Pimm, 1987) - when they have to organize educational strategies in the linguistic field, and consequently not all of them dedicate time to such significant activity, but prefer to delegate the collegues of Arts to do this. Experience (not just our experience) confirms however the efficacy of a working method that favours the metacognition and the linguistic reflection and its high potentiality - even psychological - in relation to the acquisition by the students of a greater confidence in themselves, in the schoolmates and teachers.

CONCLUSIVE CONSIDERATIONS

We shall conclude by underlining some open aspects of this work.

A first one concerns the analysis of different and articulate reasons which could be at the basis of the attitudes of students following the *Rosella-model* (more generally, the D/E students), namely at the basis of their generalized *difficulties* in front of logical problematic situations: e.g. the social-cultural conditioning, a doubtful or unsuccessful didactic agreement (see Brousseau, 1986) and so on.

An other one concernes the difficulty to value the relapses that activities like this can induce in the general linguistic competence of the students, outside such an unusual work and so precisely tending to understand, according to the situations, differences or logical-linguistic analogies even when not very evident.

It is difficult too because, in the normal didactic practice, the opportunity to reproduce situations so favourable to the linguistic reflection seldom occurs; moreover, not every teacher pays so much attention to written and oral verbalization.

It is equally complex to find out clearly the connections between the developement of the linguistic competence and the improvement of the mathematical knowledge, but many researchers (for instance Laborde, 1990) acknowledge the efficacy of verbalization and of discussions among the students on this subject, even though at the same time they underline the need of getting to the bottom of the dynamics of the linguistic interaction in the class and the knowledge of more propitious didactical contexts.

Undoubtedly in our work we have three chances, partly already revealed: - even within the syllabuses settled by the Italian government - we can fix a very flexible term in the achievement of our aims, we have been directing the two-hours lessons every week constantly together, we are willing to do a regular comparison which we could define as epistemological, between our disciplines.

This enables us to point particularly to the quality more than to the quantity of the knowledge, without excessive problems about the time necessary to find it. These are probably infrequent aspects in the ordinary school routine.

Indeed, it is necessary to dispose of long periods of time, because it is the class in its whole that conditions the developement of the activity and guides the teachers to one or another possible direction, compelling them to call the attention towards the large

variety of answers offered by the students, in order to organize (often in short time of reflection) the operating outline of the work to be developed, and to propose consequently situations more suitable to clarify or adjust concepts often not easily analyzed also because of the frequent ambiguities implicit in the natural language.

In other words it is a matter of a kind of work in which a *circular relation* develops between an accurate analysis of the situation to be proposed and of its possible expansions and the critical interpretation of the student's attitudes and of the protocols created by them *in itinere*.

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